Forgetful Consumers and Consumption Tracking Costs *

Ying Bao† Peter Landry‡ Mengze Shi§

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†Rotman School of Management, 105 St George St, Toronto, ON, Canada M5S 3E6. Email: ying.bao14@rotman.utoronto.ca.

‡University of Toronto, Institute for Management and Innovation and Rotman School of Management, 3359 Mississauga Road, Suite 2200, Mississauga, ON, Canada L5L 1C6. Email: Peter.Landry@rotman.utoronto.ca

§Rotman School of Management, 105 St George St, Toronto, ON, Canada M5S 3E6. Email: Mshi@rotman.utoronto.ca.

Abstract

Penalty fees for consumption overage are a ubiquitous pricing practice in many service industries including wireless communication and retail banking. Despite the technologies such as mobile apps that companies have developed to reduce consumer's cost to track the consumption, penalty fees remain a common practice and industries still report steady revenues from penalty fees. This paper examines the marketing implications of the advances in consumption tracking technologies. Towards this end, we develop a two-period consumption model in which a firm offers a service contract that consists of a subscription fee and a penalty fee. Paying the subscription fee gives consumers the access to service in each of two periods. Consumers must pay a penalty fee if they consume in both periods. Consumers can be "forgetful" in the sense that, upon reaching the second period, they may not perfectly remember whether they consumed in the first period. Moreover, while consumers are forward looking in the first period, they may not be "sophisticated" enough to perfectly anticipate the extent of their forgetfulness in the second period. Our analysis focuses on a market of consumers having a homogeneous level of forgetfulness and sophistication. In such a market, if consumers do not have the option to track their first-period consumption, the firm will charge a penalty fee when consumers are naïve; the equilibrium penalty fee would be higher when consumers exhibit a lower level of sophistication. With the advancement of consumption tracking technology, we find that the availability of tracking option prompts the firm to reduce its penalty fee to disincentivize consumers from using it. In the equilibrium, it can be rational for consumers to be forgetful (do not track) and the penalty fees persist. This paper also examines a market where consumers belong to two segments with heterogeneous levels of forgetfulness. Our analysis shows that depending on the distribution of consumers in two segments (more vs. less forgetful), some consumers can be worse off when the tracking cost decreases. Specifically, a reduction of penalty fee can lead the less-forgetful consumers to pay more penalty fees. These results confirm the importance for the firms and regulators to consider consumers' limited sophistication towards their forgetfulness in the respective pricing and policy decisions. The paper also provides a new model of consumer sophistication and offers a theory to explain consumer's rational inattenton through a firm's endogenous price adjustment in response to technological changes.

Keywords: forgetting, sophistication, pricing, consumption tracking technology

1 Introduction

Over the past decade, penalty fees have become an increasingly important component of such industries as health care, cellular services and retail banking. For example, focusing on overdraft fee, which incurs when a consumer overdraws his/her checking account, the Consumer Financial Protection Bureau (CFPB, 2017) documented that U.S. consumers paid $34.7 billion in overdraft fees in 2017. When the Financial Conduct Authority (FCA) asked British consumers reasons for incurring overdraft fees, they found that only 7% of consumers in the survey indicated that they "knew it would happen but had to make a payment". Additionally, most stated they did not keep track of their transactions/spending and erroneously believed that they had enough money in their account (FCA, 2008). In addition to this, the FCA (2018) documented that a large proportion of over-drafters (28%) incur fees on transactions worth less than 10 pounds, which is equivalent to a daily interest rate of at least 20%, much higher than the cash advance rates offered by credit cards. Ample evidence of similar findings is available, suggesting that a large proportion of overdraft fees are incurred by mistake and could have been avoided had consumers been more aware of their financial situation (Stango and Zinman, 2014).

Consumers could track their prior consumption by writing down every transaction they have made, which is costly for most people to do that on a regular basis. The effect of technological advancement is to reduce the cost for consumers to track their prior consumption. In particular, the advancement in technology can help consumers be more aware of their past consumption and avoid paying careless penalty fees more easily[1]. For example, consumption histories can be checked using mobile applications rather than having to wait in line at a bank branch for information before making a purchase. Some researchers (Grubb and Osborne, 2015; Grubb, 2015) have suggested that if consumers could access timely and convenient information regarding their current bank balances, the prevalence of overdraft fees would be eliminated. However, penalty fees remain ubiquitous; in other words, the advances in tracking technology have not seen clear reduction of tariff. Little is known about firms' responses and consumer welfare concerning the advancement

[1] https://www.forbes.com/sites/bernardmarr/2017/02/10/a-complete-beginners-guide-to-fintech-in-2017

of consumption tracking technology.[2]

This paper studies the effect of consumption tracking technology on market behavior by using a two-period consumption model, wherein a monopoly firm in the market provides a service contract for an initial sign-up price that allows consumers to use the service over two distinct periods, and a penalty fee is applied if consumers choose to consume in both periods. In the model, consumers can have imperfect memory regarding their period-one decision which will affect their period-two decision. Ericson (2011) documented that consumers may not be fully aware that they are forgetful and be overconfident about their prospective memory. In this paper, we allow consumers in period one to have varying levels of understanding about how forgetful they will be in period two. We use the term *sophistication parameter* (μ) to describe how sophisticated consumers are regarding their memory. Consumers in this model make consumption decisions based on their beliefs about their past and future forgetfulness. In addition to using their perception, consumers also have the option to track their period-one consumption at some cost. The advancement of tracking technology in this paper can also be understood as the reduction in the cost of tracking one's consumption history.

This paper premise begins with a market of consumers having homogeneous levels of forgetfulness and perception, and demonstrates that in the benchmark case where consumers do not have the option to track their first-period decision, the firm will charge consumers with lower levels of sophistication a higher penalty fee. However, the advancement of consumption tracking technology prompts the firm to weakly reduce its penalty fee to disincentivize consumers from using it. Thus social welfare improves despite the firm's profit reduction. Later, We use the model to assess the effect of consumption tracking technology by allowing consumers to be heterogeneous in their levels of forgetfulness. In particular, if the proportion of less forgetful consumers is large, the firm will decrease the penalty fee and the advancement of tracking technology can instead hurt less forgetful consumers since they must pay more on penalty fees. Meanwhile, the firm's profit does not change much owing to the revenue earning charges levied against existence of less forgetful consumers. Otherwise, the firm might only target more-forgetful consumers by increasing the penalty

[2]https://www.heraldtribune.com/news/20190304/john-hielscher-state-taps-new-regulator-for-financial-services-providers

fee. This is because the firm will design its contract to exploit those more forgetful consumers and a lower tracking cost serves as a tool for the firm to discriminate between consumers.

One closest research on this topic was carried out by Grubb (2015). He studied the market with penalty fees when consumers are forgetful (inattentive) about their previous activity and concluded that the penalty fees result from consumers who naively believe they are attentive and underestimate the probability of incurring penalty fees. However, he only studied consumers who naively believe that they are attentive and thus have no need to track their past behavior. However, Ericson (2011) suggests that consumers are partially forgetful, and do not have precise knowledge about their forgetfulness. It is more realistic to assume that most consumers are some blend of forgetful and sophisticated. Moreover, some important behavior can be captured by including those consumers. For instance, unlike completely naive consumers, partially naive consumers might consider the cost of using tracking technology while planning their consumption and purchasing decisions and have a lower willingness to pay for the contract. This will likely result in their leaving leave the market unless the firm changes the contract. To prevent those partially naive consumers from leaving, the firm is likely to increase the sign-up price and eliminate the penalty fee.

Another key difference is that Grubb (2015) focuses on the effect of "bill-shock" regulation (modeled as penalty fees ban), our model instead studies the effect of advancements made in consumption tracking technology and allows the firm to re-design its service contract freely in response. Results indicate that a reduction in tracking cost will weakly decrease the penalty fee and weakly increase the sign-up price. Furthermore, the consumption tracking technology could result in less forgetful consumers paying more on penalty fees.

This paper has important implications with respect to the policies designed to reduce tracking costs. Regulators have focused on the amount of penalty fees paid by consumers due to avoidable mistakes and high tracking costs is believed to hinder consumers from tracking their account. Regulatory authorities have attempted to remedy this situation by reducing the tracking cost. For instance, the UK government required banks to give their customers access to a suite of overdraft alerts by text message in 2012 (FCA, 2015). In 2018, the regulatory authority went even further,

requiring banks to automatically enroll consumers into programs that would send them alerts when they are about to incur an overdraft fee (FCA, 2018). Nevertheless, prior to the auto-enrollment program, very few customers (3% - 8%) had registered for alerts services provided by the UK banks (FCA, 2015). The paper provides an explanation for this: consumers do not believe they need alerts. Moreover, when the government agency evaluated the effectiveness of those policies, they did not take the firm's response into account. The results from then paper confirm the importance for the firms and regulators to consider consumers' lack of sophistication towards their forgetfulness. In particular, it suggests that the policy's effectiveness depends on the extent of the reduction in tracking cost. A minor reduction might have no impact at all- it may even impact some consumers negatively. If the tracking cost can be driven to a very low level, the firm will respond by setting the penalty fee to zero while initiating a high sign-up price. As such, social welfare will improve despite a reduction in the firm's profit.

The results of our study are organized as follows. Section 2 reviews related literature. The benchmark model with no consumption tracking option is introduced in Section 3, where the firm's optimal pricing strategy for different types of consumers is discussed. Section 4 investigates the effect of consumption tracking technology, including the consumers' tracking decisions and the firm's response and then analyzes any changes in social welfare. Section 5 focuses on the market share made up of consumers with heterogeneous levels of forgetfulness, and Section 6 examines potential policy implications. Our conclusion includes a discussion of potential research opportunities stemming from the modelling done in this paper.

2 Literature Review

Consumers have been reported to pay a significant amount of money on penalty fees in industries like retail banking, cellular services and health insurance and it generates considerable controversy and policy scrutiny (FCA, 2008, 2018; CFPB, 2017). Survey evidence suggests that consumers incur bank overdraft fees due to mistakes such as they do not know their account balance and believe they have enough money in their account cover the expenditure (FCA, 2008). It suggests

that consumers cannot perfectly recall the past, and that they make a consumption based on some heuristics that lead to a sub-optimal allocation.

This paper is related to the literature about imperfect memory. In the fields of psychology and experimental economics, researchers have documented significant evidence for the fact that people can not accurately recall the past (such as the paid price for goods and services) due to memory capacity constraints (e.g.,Miller 1956; Dickson and Sawyer 1990; Helgeson and Beatty 1987), and that they will use their own judgment/perception to make decisions. In addition, a number of researches have suggested that people both engage in selective search and recall, and that they tend to overestimate the probability of positive events (Greenwald, 1980; Crocker et al., 1984; Mitchell et al., 1997; Li, 2013). Although the literature indicates ample attention has been paid to memory from consumer's perspective, little research has focused on how the presence of limited memory affects a firm's pricing and market equilibrium. Among the few examples, Rubinstein (1993) analyzes a model in which a firm price discriminates between consumers who have different memory capacities by choosing a random lottery of prices. Chen et al. (2010) , focus on limited memory and price. In particular, consumers in their paper used categorization process to make inferences. They demonstrated that the presence of limited-memory consumers along with the perfect-memory consumers can soften price competition between the firms. Moreover, across all different categorization processes, the expected profits of the firms always increase when there are a greater number of uninformed consumers. Our paper models imperfect memory by focusing the impact that past decisions have on current decisions and that process's impact on the firm's pricing and market equilibrium.

This paper also incorporated literature regarding consumer inattention, particularly on inattention to the past activities, which can also be interpreted as forgetting [3]. The fact that consumers do not keep track of past usage and are generally inattentive to their past behavior is fairly common in some industries. For example, Grubb and Osborne (2015) documented evidence for consumer inattention and structurally estimate a model of inattention in the cellular services industry. In

[3] From communication process perspective, people encode the information first, then process, and decode the information. Although forgetting and inattention differs from each other in the sense of whether people have encoded the information, the two concepts share the similarity that people are not able to retrieve the information. In the paper, we focus on the similarity.

particular, they found that consumers are inattentive to their own past usage and underestimate their own uncertainty about future calling demand. In the financial industry, Liu et al. (2016) found evidence for consumer inattention and provide empirical evidence for rational inattention: high tracking costs hinder consumers' effort to track their balance accurately. When Alan et al. (2018) conducted a field experiment in Turkey, they found evidence that consumers are inattentive to overdraft costs, and firms will shroud overdraft prices as response in equilibrium. Overall, the empirical evidence from past research suggests that consumers will often not remember about their past transactions or usage. Grubb (2009) showed theoretically that consumer overconfidence about future usage explains the multi-part tariff plan in the cellular phone service industry. Grubb (2015) later developed a model of dynamic inattentive consumption and found that naive consumers will underestimate the probability of being charged a penalty fee, a factor which leads to the existence of penalty fees. However, he did not consider consumers who can be partially inattentive and partially naive. Our research develops a generalized model of dynamic inattention; We also explicitly model consumer decisions about using the consumption tracking technology and consider the firm's response when evaluating the effect that the advancement of consumption tracking technology has on market equilibrium.

3 Model

We model a market with a monopoly firm and a unit mass of consumers.[4] The contract provided is a multi-part tariff with the following feature: consumers pay a sign-up price, p [5] that allows them to use the service over two periods, as well as a penalty fee, ϕ, that applies when consumers use the service (modeled as consumption decision) in both periods.

Consumers make three types of decisions. The first involves whether to purchase the service contract provided by the firm.[6] We abstract from the reality and further assume that there are two

[4] According to the report by FCA (2015), most consumers do not switch accounts among banks based on overdrafts fees paid, implying that there is little pressure on providers to compete. As a result, each bank can be seen as a local monopolist. TheFCA (2008) documented similar findings.

[5] One can interpret the sign-up price as the monthly fee associated with checking accounts, cellular phone plans

[6] If they choose not to buy the service, they will receive his/her outside value which is normalized to 0

consumption periods. Conditional on purchase (paying p), in period $t \in \{1,2\}$, consumers will then make consumption decisions over two periods. If they consume in both periods, they will be charged a penalty fee ϕ. At $t = 2$, but prior to their consumption decision, consumers can decide whether or not they want to use track their consumption at a cost k[7]. Tracking their consumption allows them to determine in real time whether they have consumed in period one or not. The timing of consumers' decision summarized is shown in Figure 1:

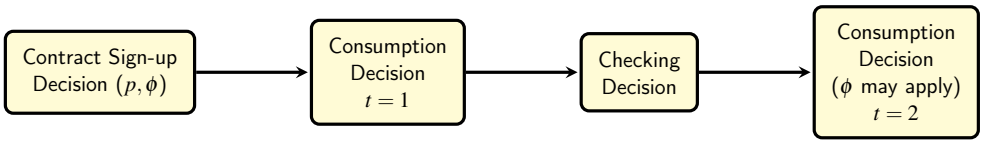

Figure 1: Timing

At each period $t \in \{1,2\}$, they privately receive a value v_t which generates the opportunity to consume, following which they make their consumption decision. We assume that the shock v_t is drawn independently from a Uniform Distribution $U[0,1]$. Hence, at period two, a consumer's realized total utility can be expressed as:

$$U = -p + v_1 d_1 + v_2 d_2 - \phi d_1 d_2$$

where $d_t \in \{0,1\}$ is denoted as a consumer's consumption decision in each period. Here, We depict the consumption decision as binary and $d_t = 1$ if consumers choose to consume in period t. The equation above suggests that consumers pay a sign-up price (p) to enjoy the service over two periods, but if they consume in both periods, they will be charged the penalty fee ϕ.

In each period, consumers make their consumption decision by comparing the realized value v_t with a threshold ($\underline{v_t}$) which makes consumers indifferent from consumption. Given the distribution of v_t, We are able to express the consumption probability in each period. The way how consumers make their consumption decision is called the threshold rule, and it has been shown to be optimal and is used extensively in the literature (e.g., Grubb, 2015; Grubb and Osborne, 2015).

[7]The cost is called tracking cost in the paper. It can interpreted as monetary, or psychological cost.

Here, consumers are assumed to be forward-looking and to plan their consumption decisions ahead of time. In other words, consumers take the probability of being charged a penalty fee into account when they make consumption decisions. It is notable that the discount factor is assumed to be 1. A key feature of the model is that consumers' second period utility/decision depends on their period one decision. The threshold $\underline{v_t}$ for period t depends on consumers' anticipation regarding their consumption decisions in the next period(s). More specifically, when consumers make their period-two decision, they rely upon their memory about their period one consumption decisions. We allow that consumers' memory will be imperfect and that there is some probability that they forget their period-one decision. Previous research suggests that consumers have a significant tendency to forget their consumption history (Greenwald, 1980; Crocker et al., 1984; Mitchell et al., 1997; Li, 2013).[8] To capture the asymmetry described in literature, we further assume that if consumers do consume in period one, they might not accurately remember doing so in period two. We use α to represent the uncertainty that consumers mistakenly believe that they did not consume in period one. It is a measurement of consumers' memory capacity constraints that can also be interpreted as how forgetful consumers are.

When consumers make their purchase and period-one decisions, they incorporate their prediction about future consumption based on their perception. Another feature of the model is that it allows consumers to have varying levels of perceptions about their forgetfulness. In this model, $\tilde{\alpha}$ is denoted as the consumers' belief at period one regarding α. We also assume that $\tilde{\alpha} \leq \alpha$ in the model, implying that consumers are under-confident about their memory.[9] This assumption is consistent with Ericson (2011), which documented that consumers might not be fully aware of their forgetfulness and thus tend to underestimate their forgetfulness.[10]

[8] In the psychology literature, researchers have extensively studied memory accuracy and found that people tend to remember events asymmetrically. Recently, some theoretical models and empirical evidence in economics (Greenwald, 1980; Crocker et al., 1984; Mitchell et al., 1997; Li, 2013) have suggested that people engage in selective search and recall, and thus overestimate the probability of positive events.

[9] If $\alpha < \tilde{\alpha}$, it suggests that consumers are under-confident about their memory; in other words, they believe that they are more forgetful than they actually are. We show that it is always optimal for the firm to provide a contract with 0 penalty fee and a high sign-up price ($p = 1$).

[10] It is notable that we can also model this as follows: in the market, there are α proportion of consumers who always forget they have consumed in period one and the rest $1 - \alpha$ consumers who always remember their period one decision. Moreover, those consumers with imperfect memory hold some belief $\tilde{\alpha}$ about their forgetfulness. The qualitative results remain the same in either case.

Even those consumers who are weakly overconfident about their memory, they can be categorized into sophisticated forgetful consumers who understand about their memory limitations and naive forgetful consumers who mistakenly believe they have better memory than do. To better understand consumer types, We introduce a sophistication parameter $\mu := \frac{\tilde{\alpha}}{\alpha}$ to capture the extent to which consumers are aware of their forgetfulness. The lower μ is, the more naive the consumers is regarding the extent of their forgetfulness. When forgetful consumers incorrectly believe that they have perfect memory, they will have $\mu = 0$.

Section 3 examines how a firm can best optimize the service contract provided to each type of consumer in response to their differing levels of demand. Although consumers with near-perfect memory is are not the focus of the model, understanding how they make decisions illuminates key market mechanisms. In Section 3, the tracking cost k can be envisioned as being large enough that consumers choose not to track their consumption history. We will relax the assumption in Section 4 to examine firm's pricing and consumer welfare when tracking costs is lower due to technological advancement.

3.1 Consumers with Perfect Memory

Consumers with perfect memory are those who understand that they will remember their activities from previous period ($\tilde{\alpha} = \alpha = 0$). To determine the contract for those consumers, We first establish the demand by using backward induction.

When consumers reach period two, if they consumed in period one ($d_1 = 1$), they will choose not to consume again unless the value they get is higher than the penalty fee. That is, if $\phi \leq 1$, given the distribution of v_t, the probability of consuming in period two is $\Pr(d_2 = 1) = 1 - \phi$. As a result, if $d_1 = 1$, consumers' expected utility from period two becomes $\frac{(1-\phi)^2}{2}$. However, if they did not consume in period one, consumers will always choose to consume in period two, given the absence of a penalty fee in this case. As a result, if $d_1 = 0$, the expected utility from period two is $\frac{1}{2}$. In period one, those consumers will choose to consume if and only if the total expected utility they get from consuming in period one is at least as high as the value they get from not consuming in period one. Thus, the threshold that determines consumer's consumption decision for period one

is $\underline{v_1} = \phi - \frac{\phi^2}{2}$. When facing a penalty fee greater than 1, consumers will never choose to consume in period two if they have consumed in period one. In this case, if $d_1 = 1$, the expected utility from period two is 0 and the threshold for consuming in period one is $\underline{v_1} = \frac{1}{2}$. This suggests that when the penalty fee is too high, consumers will only consume once over both periods, but having the option to choose allows consumers to wait unless they get a particularly high value from period one.

The probability of consuming in period one can thus be calculated as $\Pr(d_1 = 1) = 1 - \underline{v_1}$, and the threshold for consuming in period one can be written as follows:

$$\underline{v_1} = \begin{cases} \frac{1}{2}, & \phi > 1 \\ \phi - \frac{1}{2}\phi^2, & \phi \leq 1 \end{cases} \tag{1}$$

When consumers make purchasing decisions, they consider their consumption strategy in each period. As expected, a higher penalty fee reduces consumer's willingness to pay for the service contract. The firm's profit comes from two sources: the sign-up price (p) and revenue from penalty fees ($d_1 d_2 \phi$). To maximize its expected profit, the firm will extract expected consumer surplus by setting the sign-up price at the level of consumer's willingness to pay. The probability of being charged a penalty fee, which is $Pr(d_1 = d_2 = 1) = (1 - \underline{v_1})(1 - \underline{v_2})$, decreases along with the penalty fee. By maximizing the firm's expected profit, We am able to determine the optimal contract for consumers with perfect memory as follows:

LEMMA 1: *For consumers with perfect memory, it is optimal for the firm to charge a high sign-up price ($p^* = 1$) and not to charge a penalty fee ($\phi^* = 0$). In equilibrium, consumers will consume in both periods. Moreover, this contract maximizes social welfare.*

The rationale for this pricing scheme is that when the firm sets the penalty fee at a positive level, consumers with perfect memory will consider that fee when they choose their consumption strategies; they thus have a lower willingness to pay. Although the revenue from penalty fees might increase, the firm needs to lower the sign-up price to make sure that consumers will purchase the contract. However, the revenue from penalty fees cannot make up the losses due to lowering the

sign-up price. As a result, the optimal strategy is for the firm charge a high sign-up price ($p^* = 1$) and remove the penalty fee altogether($\phi^* = 0$). The resulting level of profit earned by the firm is $\Pi^* = 1$. In equilibrium, consumers' expected and realized utility after paying the sign-up price is 0, and the total surplus is 1. Indeed, from the social planner's perspective, this contract maximizes social welfare since consumers are able to consume in both periods and the firm maximizes its profit.

3.2 Forgetful Consumers

Forgetful consumers are defined as those who are not able to accurately recall their period-one decision when they reach period two. They thus rely on their perception when making consumption decisions. We follow the same procedure as in Section 3.1 to examine how those forgetful consumers make consumption decisions. In period two, forgetful consumers realize their forgetfulness and believe that they have $(1-\alpha)$ chance to pay a penalty fee, and they make their consumption decision by weighing the realized value against expected penalty fee to pay ($(1-\alpha)\phi$). Depending on consumers' perception about how forgetful they are, the threshold for consumption in period one is determined by making consumers indifferent regarding whether they should consume in period one or not.

Figure 2 shows how consumers with different perceptions about their forgetfulness behave in period one when they face different penalty fees. Consumers who choose to consume in period one can be usefully classified into three types: confident consumers who expect not to pay a fee; reluctant consumers who expect to pay a partial fee; and scared consumers who expect to pay the full penalty fee.

When the penalty fee ϕ is small, then regardless of their perceptions, consumers who believe that they have some probability of incurring a penalty fee will adjust their consumption decision (threshold) based on their perceptions and penalty fee amount. Those consumers are called reluctant consumers.

When the penalty fee is significant, a consumer's consumption strategy for period one depends on their perception of future forgetfulness: For a given level of penalty fee, as $\tilde{\alpha}$ increases,

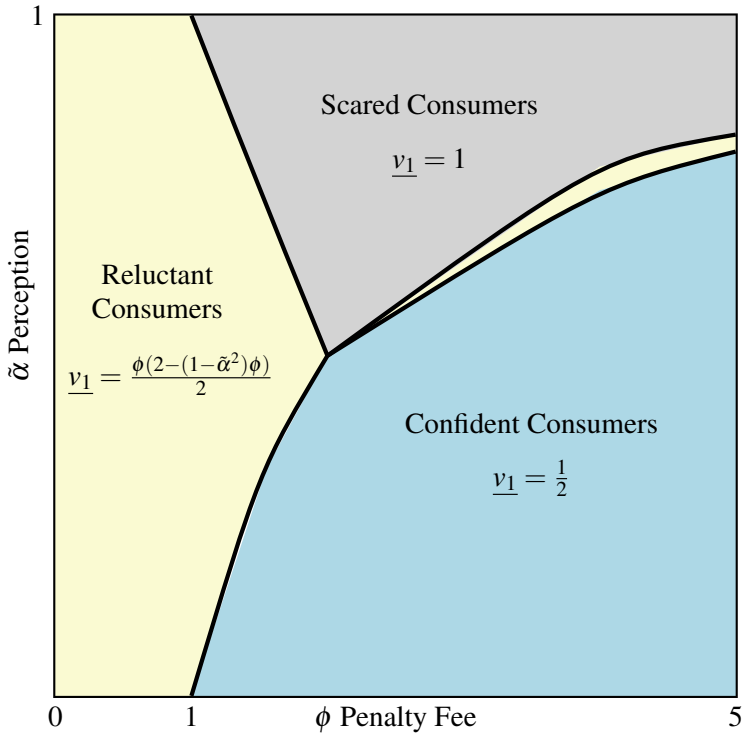

Figure 2: Period one Consumption Decision

the probability of consuming in period one decreases from $\frac{1}{2}$ to 0. More specifically, in period one, confident consumers believe that their memory is accurate enough that they naively believe that they will never incur a penalty fee. In other words, they plan to consume only once over both periods. Having the option to choose between two periods prompts consumers to be more selective when making their period-one decision. In particular, they will consume in the first period if the value they get is higher than $\frac{1}{2}$. Reluctant consumers partially understand their memory limitations, and account for the possibility of incurring a penalty fee; they become less likely to consume in period one. Scared consumers, on the other hand, believe they will forget and incur a penalty fee if they consume in period one. In order to avoid paying high penalty fee completely, those consumers would rather forgone the opportunity to consume in period one. Overall, consumers are more likely to consume in period one when the penalty fee is larger and when those consumers underestimate their forgetfulness.

Consumers' willingness to pay for the service contract relies on their perception about their

future forgetfulness and the level of the penalty fee. Their willingness to sign up and pay for the consumption tracking service can be written as follows:

$$E(\tilde{U}) = \begin{cases} \frac{5}{8}, & \tilde{\alpha} \leq 1 - \frac{1}{\phi} \\ 1 - \phi + (1 - \frac{\tilde{\alpha}^2}{2})\phi^2 - \frac{1-\tilde{\alpha}^2}{2}\phi^3 + \frac{(1-\tilde{\alpha}^2)^2}{8}\phi^4, & 1 - \frac{1}{\phi} < \tilde{\alpha} \leq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \\ \frac{1}{2}, & \tilde{\alpha} > \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \end{cases} . \quad (2)$$

The first line in Equation 2 describes the perceived expected utility for the group of confident consumers depicted in Figure 2. Those consumers believe that they will never incur a penalty fee, which also suggests that they plan to consume only once over the two periods. As mentioned earlier, having the option to choose to consume between period one and two offers them extra value because they are able to wait till the next period if the value is lower than $\frac{1}{2}$ in period one. As a result, although the consumers only plan to consume in one period, the perceived expected utility is greater than $\frac{1}{2}$ and reaches $\frac{5}{8}$. The second line in Equation 3 illustrates the perceived expected total utility for Figure 2's reluctant consumers. As expected, the more naive forgetful consumers tend to overestimate the utility from the contract/service. The third line of Equation 2 represents scared consumers from Figure 2; they will only consume in period two, thus their expected utility will be $\frac{1}{2}$.

The firm's profit, which can be expressed as $\Pi = p + d_1 d_2 \phi$ are are derived from both the sign-up price and from penalty fees. As a monopoly, its optimal strategy is to extract all the surplus possible from those forgetful consumers, which means the sign-up price should be set to as high a price as consumers are willing to pay. Given the consumption strategies described previously, the actual/realized probability of incurring a penalty fee can be written as follows:

$$Pr(d_1 = d_2 = 1) = \begin{cases} 0, & \alpha < 1 - \frac{1}{\phi} \text{or } \tilde{\alpha} > \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \\ \frac{1-(1-\alpha)\phi}{2}, & \tilde{\alpha} \leq 1 - \frac{1}{\phi} \leq \alpha \\ (1 - \phi + \frac{1-\tilde{\alpha}^2}{2}\phi^2)(1 - (1-\alpha)\phi), & 1 - \frac{1}{\phi} < \tilde{\alpha} \leq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \end{cases} . \quad (3)$$

If the penalty fee is so high that the expected penalty fee $(1-\alpha)\phi$ is greater than 1, consumers will never choose to consume in period two; there is then no probability that a penalty fee will be incurred. As a result, part of the condition for the first line is $\alpha < 1 - \frac{1}{\phi}$. The scared consumers depicted in Figure 2, who avoid consuming in period one and and will only consume in period two, will also have zero probability of incurring a penalty fee. The second line in Equation 3 describes instances when consumers expect to pay no penalty fee in period one, but might incur penalty fees when they reach period two. As describe in Figure 2, those confident consumers have $\frac{1}{2}$ chance of consuming in period one, which is independent of their perception. While their second period decision depends on their forgetfulness. It explains why the equation is independent of $\tilde{\alpha}$. The overall probability of incurring a penalty fee increases for more forgetful consumers. The third line of the Equation 3 represents reluctant consumers who take the positive probability of incurring a penalty fee into account. In particular, the probability of incurring a penalty fee increases as those consumers become more forgetful (α) or believe they have more accurate memory ($\tilde{\alpha}$). With respect to the potential revenue generated by penalty fees, the firm earns greater profits by exploiting more forgetful or naive consumers.

Liu et al. (2016) found that frequent over-drafters tend to spend more at the beginning of a pay period than at the end of a pay period, while infrequent- or never-over-drafters tend to have a stable spending pattern,. This model provides an explanation for this empirical finding. In our model, more naive consumers are more likely to consume in period one. Moreover, the difference between consuming in period one and period two is larger for consumers with lower $\tilde{\alpha}$. As discussed previously, consumers who have a lower $\tilde{\alpha}$ are more likely to incur a penalty fee. Taken together, then we see a higher probability that the frequent over-drafters will consume in period one and that the difference in spending patterns between the two periods is larger.

Turning to the optimal contract, scared consumers from region III in Figure 2 who face a high penalty fee and have a high $\tilde{\alpha}$, the firm has to set the sign-up price to be $\frac{1}{2}$ to encourage those consumers to participate, but it then makes no revenue from penalty fees. In this case, the firm only earns a profit of $\frac{1}{2}$ from the sign-up price. Hence, it should never choose to charge those consumers a high penalty fee for those consumers. The firm should rather set a high sign-up

price and assess no penalty fees. For reluctant consumers who partially understand their memory limitation, their willingness to pay and the actual probability of incurring a penalty fee depends on their perception $\tilde{\alpha}$, particularly when the penalty fee is not so large that they believe they could not afford it ($(1-\tilde{\alpha})\phi < 1$). On the other hand, confident consumers will believe that they will not incur a penalty fee and their willingness to pay for the service contract is $\frac{5}{8}$. In either case, consumers have positive probability of incurring a penalty fee.

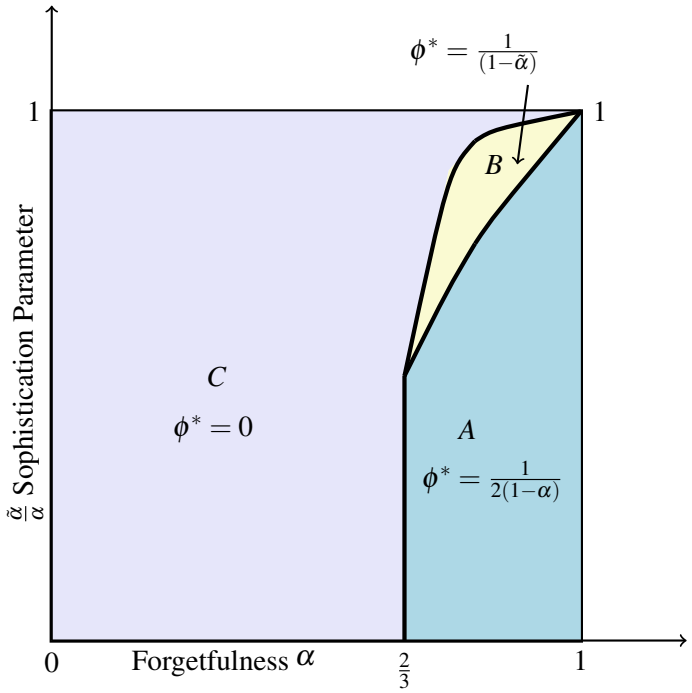

Figure 3: Regions in Optimal Contract

The optimal contract is one that maximizes firm's expected profit. To better understand the optimal contract, We use Figure 3 to illustrate how the contract applies to consumers with different levels of forgetfulness and perception. To better interpret the results, We re-write the conditions in terms of forgetfulness and sophistication parameter. In that figure, Regions A, B and C represent the three contract types. First, when consumers are not sufficiently forgetful ($\alpha < \frac{2}{3}$), the firm should always provide a contract with a high sign-up price and zero penalty fee, regardless of how sophisticated they may be. The reason is that the low level of forgetfulness reduces the gain from penalty fee so much that they will not cover the losses incurred by having

a reduced sign-up price. Second, when consumers are sufficiently forgetful $\alpha \geq \frac{2}{3}$, the contract will depend on how sophisticated consumers (μ) are. In general, consumers from Region A tend to be less sophisticated, consumers from Region B are moderately sophisticated, and Region C consumers are highly sophisticated. As expected, the levels of penalty fee reflects the same order: $\frac{1}{2(1-\alpha)} > \frac{1}{1-\tilde{\alpha}} > 0$.

In Region A, where the sophistication parameter is small ($\mu \leq 2 - \frac{1}{\alpha}$ and $\alpha > \frac{2}{3}$), the firm charges a penalty fee $\phi^* = \frac{1}{2(1-\alpha)}$, which increases for more forgetful consumers. Since those consumers believe that they will not incur a penalty fee, the firm will set the sign-up price as their willingness to pay ($p^* = \frac{5}{8}$). Indeed, those consumers fall within the category of confident consumers depicted in Figure 2. In Region B where the sophistication parameter increases and falls within the range ($2 - \frac{1}{\alpha} < \mu < \frac{\alpha+2\sqrt{3\alpha-2}}{3\alpha}$), consumers become more sophisticated and take the positive probability of incurring a penalty fee into account, thus their willingness to pay depends on the penalty fee and perception as well. In this case, the profit is increasing in penalty fee. Therefore, the firm will charge $\phi^* = \frac{1}{1-\tilde{\alpha}}$ to exploit consumers who consider the positive possibility of incurring a penalty fee. Those consumers are corresponding to the reluctant consumers in Figure 2. While for Region C where $\alpha > \frac{2}{3}$, a higher sign-up price and zero penalty fee using a rationale similar to that explained previously: the high sophistication parameter indicates that scared consumers are relatively more aware of their forgetfulness and thus less likely to incur a penalty fee, resulting in less revenue from penalty fees that is smaller than the amount that the sign-up price was reduced to ensure consumers will purchase the service contract.

Therefore, We can summarize the findings in Proposition 1.

PROPOSITION 1: *It is optimal for the firm to charge (i)* $p^* = \frac{5}{8}$ *and* $\phi^* = \frac{1}{2(1-\alpha)}$ *if* $\alpha \geq \max(\frac{1+\tilde{\alpha}}{2}, \frac{2}{3})$*; (ii)* $p^* = \frac{5}{8}$ *and* $\phi^* = \frac{1}{1-\tilde{\alpha}}$ *if* $\frac{3\tilde{\alpha}^2-2\tilde{\alpha}+3}{4} < \alpha \leq \frac{1+\tilde{\alpha}}{2}$*; (iii) otherwise,* $p^* = 1$ *and* $\phi^* = 0$*. Any positive penalty fee reduces consumer surplus and total welfare.*

In equilibrium, consumers from Region C pay a high sign-up price ($p = 1$) and do not need to pay penalty fees if they consume in both periods. Therefore, they will choose to consume in both periods to maximize their expected utility, and the firm's profit is generated by the higher sign-up price. The firm charges consumers from Region A and B the same sign-up price but assesses

different penalty fees depending on their sophistication parameter. Consumers from Region A and B behave differently in equilibrium. In particular, consumers from Region A expect 0 probability of being charged a penalty fee, but in reality, they have a $\frac{1}{4}$ probability of consuming in both periods. Although consumers from Region B have the same probability of consuming in period one as those from Region A, they tend to have a lower probability of consuming in period two. Overall, the probability of incurring a fee and the potential revenue from penalty fees paid by consumers from Region B are lower since they are more sophisticated than those from Region A.

As mentioned in Section 3.1, from a social planner's point of view, the firm should set the penalty fee to be 0 and consumers should consume in both periods. Any positive penalty fee would decrease the consumers' welfare since the penalty fee makes consumers less likely to consume in both periods. Here, the firm extracts all the consumer-perceived surplus, and the positive penalty fee which results from exploiting consumer bias (overconfident about how forgetful they are) leads to a negative realized consumer surplus. Although the firm profits more from contracts with positive penalty fees, the total welfare decreases. This suggests that charging a penalty fee for consumers not only hurts consumers, but also harms society as a whole.

4 Model Analysis with Low Consumption Tracking Costs

In many usage situations today, consumers are given the option to track their consumption. For example, before making a purchase, consumers can go to a local bank branch to find out what their prior consumption decisions were. However, technological advances over the past couple of decades have transformed many aspects of daily life. In particular, tracking technology provides consumers with the ability to easily track their finances by installing mobile applications from banks as well as other personal finance-tracking providers on their smart phones. Such technological advances significantly reduce the cost of tracking one's consumption.

While tracking technology and these text services have been available for several years, their impact has been limited. Questions regarding when and who will utilize the option, how the firms could/should respond and how consumer's welfare would be affected remain unanswered. In this

section, our model first allows consumers the option of using tracking technology and then focuses on the firm's response and corresponding consumer welfare to the tracking option. Indeed, Section 3 can be seen as a special case where the cost of tracking is sufficiently high that consumers will never choose to track their consumption.

4.1 Consumers' Tracking Decision

In this section, We relax the assumption that the cost of tracking one's consumption history is too high for consumers because the wide advancement of consumption tracking technology generally reduces the cost of tracking one's consumption. In the following analysis, We focus on instances where forgetful consumers know about the existence of the tracking technology. As a result, they understand that consumption tracking technology can tell them exactly what happened in period one such that they can re-evaluate their consumption strategy for period two accordingly. Forgetful consumers' tracking decisions depend on how much perceived utility they expect under tracking condition as compared to the no-tracking condition.

Overall, consumers' expectation about period-two utility relies on the updated consumption strategies and how forgetful they are, which they realize in period two. In terms of their consumption strategies, We examine the following cases: First, if consumers choose not to track their consumption, their expected penalty payment is $(1-\alpha)\phi$, and they will consume if the perceived value is above the expected cost. This is the same as when there is no tracking option. Secondly, if consumers decide to spend k tracking their period one consumption decisions, they will re-construct the expectations of utility they will get from period two. More specifically, given the conditional finding that they have consumed in period one ($d_1 = 1$), consumers will choose to consume in period two if the value they get is greater than the penalty fee ϕ; otherwise, they will choose not to consume in period two. In summary, forgetful consumers' perceived expected utility for period two can be derived based on this factor. The detailed expressions and calculations are delineated in the Appendix.

Hence, the tracking rule can be described as follows: forgetful consumers will choose to track their consumption if they receive more utility from doing so. As a result, the critical threshold

$k^*(\alpha,\phi)$ that makes forgetful consumers indifferent between tracking and not tracking can be expressed as follows:

$$k^*(\alpha,\phi)=\begin{cases}\frac{1}{2}\alpha, & \phi>\frac{1}{1-\alpha}\\ \frac{1}{2}\alpha-\frac{1}{2}(1-(1-\alpha)\phi)^2, & 1<\phi\leq\frac{1}{1-\alpha}\\ \frac{1}{2}\alpha(1-\alpha)\phi^2, & \phi\leq 1\end{cases} \tag{4}$$

A closer look at the expression of k^* suggests that as penalty fee increases, consumers are more likely to check their consumption history. It is due to the value of tracking one's consumption and avoiding pay a high penalty fee is high enough such that consumers would like to spend k. It is notable that $k^*(\alpha,\phi)$ is the threshold that consumers use when they reach period two. Prior to that, consumers generally rely on their perceptions to decide whether they want to track their consumption or not. The perceived threshold is therefore denoted as $k^*(\tilde{\alpha},\phi)$, When comparing the two thresholds, the perceived threshold is always lower than the actual, implying that consumers tend to believe that they are less likely to track their consumption. And this comes from consumer's overconfidence in their memory.

LEMMA 2: *(i) If* $\tilde{\alpha}<\sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$ *and* $k^*(\tilde{\alpha},\phi)<k<k^*(\alpha,\phi)$*, then consumers will track their consumption without planning to do so; (ii) if* $k\leq k^*(\tilde{\alpha},\phi)$*, the consumers will plan and track their consumption; (iii) otherwise, consumers will not track their consumption.*

Depending on tracking cost, there are two cases for forward-looking consumers when they choose whether or not to track their consumption history: first, if $k\leq k^*(\tilde{\alpha},\phi)$,then consumers will plan to track their consumption history and will do so when they reach period two. In this case, the probability of checking their consumption decreases with k, implying that a high tracking cost prohibits some consumers from deciding to track their consumption history. Second, if the tracking cost falls between the perceived threshold and the actual threshold ($k^*(\tilde{\alpha},\phi)<k<k^*(\alpha,\phi)$),consumers believe that they do not need to track their consumption history, but will check when they realize their memory limitation in period two. A further investigation of the tracking probability suggests that more naive consumers are more likely to change their mind and choose

to check their consumption history when they reach period two.

When considering the advancement of consumption tracking technology and the contract described in Proposition 1, consumers' checking decisions depend on the level of the tracking cost. Unsurprisingly, when the cost of tracking is high, having lower tracking costs will not determine whether consumers will choose to check their consumption history. However, when tracking costs decrease, consumers will choose to check their consumption history and are able to avoid paying the penalty fee. Overall, although tracking consumption will lead consumers to forgo some consumption opportunities, their ability to avoid paying penalty fees improves consumers' welfare.

Some empirical studies (Liu et al., 2016; FCA, 2015, 2018) have found that checking one's balance using online or mobile banking or by registering for automated text alert notifications tends to help consumers reduce the probability of paying overdraft penalty fees. The model used in this study does not differentiate between these methods; it subsumes them all into a category referred to as consumption tracking technology, and the consumers in this model use that technology. They are thus aware of their period-one consumption decisions and can afford to pay a penalty fee resulting from mistakes. As such, the probability of being charged a penalty fee is less than that of consumers who do not check their consumption history.

4.2 Pricing With Low Consumption Tracking Costs

This section analyzes consumers' consumption strategies for periods one and two as well as how the firm responds to the advancement of consumption tracking technology when designing its service contract. Consumers' awareness that they have the option to learn from their past behavior leads them to behave as forward-looking agents and re-evaluate their consumption strategies according to their perceived tracking decision. As we discussed consumers' period-two consumption strategy in the Subsection 4.1 earlier: If consumers choose to track their consumption, they will consume in period two if the value they get is higher than the penalty fee they will be charged. Alternatively, if consumers do not track their previous consumption, their period-two decision will be the same as the Section 3 consumers who do not have the option of tracking.

When consumers make their period-one consumption decisions, they will rely on their per-

ception about their forgetfulness as well as their anticipation regarding whether to take advantage of the option to track their previous consumption. If consumers expect no tracking in period two ($k > k^*(\tilde{\alpha}, \phi)$), they will make their period-one consumption without considering the cost of the tracking option. Therefore, the probability of consuming in period one will be the same as when there is no tracking option. Consumers will plan to track their consumption history if the cost is smaller than the perceived threshold ($k \leq k^*(\tilde{\alpha}, \phi)$). In that case, if ϕ is large, consumers will still believe that they will only consume once over the two periods after tracking their consumption. Hence, the probability of consuming in period one is $\frac{1}{2} - k$, where consumers account for the cost of tracking their consumption. One the other hand, if the penalty fee is small enough that consumers believe they are able to afford it, they will take the probability of being charged a penalty fee into account when they make their period-one consumption decision. As a result, the probability of consuming in period one becomes $\frac{1}{2}\phi^2 - \phi + 1 - k$. Based on both consumers' perceptions of their tracking decision and the cost of the penalty fee, We am able to estimate the probability that consumers will choose to consume in period one if they are able to track their consumption.

To better illustrate how consumers make their period-one consumption decision when the tracking costs are lower, We draw the regions for consumer's period-one decision when $k = \frac{1}{100}$. Consumers are less likely to plan to track their consumption history if the penalty fee is sufficiently low because they can afford the penalty fee by not paying the tracking cost k. On the other hand, a high penalty fee makes the expected payment from consuming in both periods so high that consumers cannot afford it, which in turn increases the benefit of paying the cost of tracking. In Figure 4, forgetful consumers (the portion surrounded by a thick red line separating Regions II and III from the rest) expect to check their consumption history in period two. As Figure 4 illustrates, consumers are more likely to track their consumption when they will incur a high penalty fee. Forgetful consumers are also less likely to consider tracking their consumption history if their $\tilde{\alpha}$ is sufficiently low or high. For example, even when faced with a high penalty fee, consumers from Region I in Figure 4 naively believe that their memory is accurate enough to remember their consumption behavior in the previous period. Meanwhile, consumers with high $\tilde{\alpha}$ consider the probability of incurring a penalty fee when making decisions regarding consumption, and they

would not need to spend k to learn d_1.

We also tried to plot similar figures by setting $k = 0$ or $k = \frac{1}{2}$ respectively. Our results indicate that all forgetful consumers will plan to check their consumption history if $k = 0$. Accordingly, if k is high, no consumers will willingly incur that tracking cost to track their consumption.

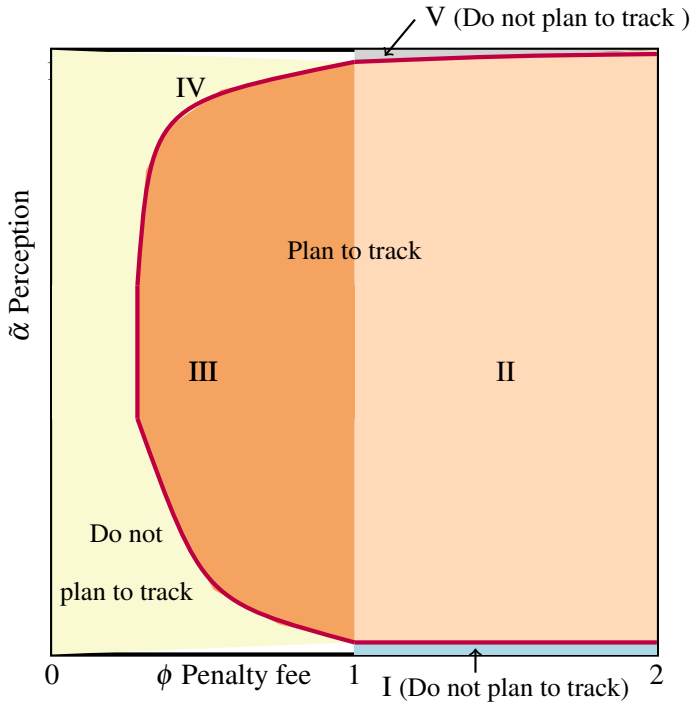

Figure 4: Regions for $\Pr(d_1 = 1)$ $(k = \frac{1}{100})$

The perceived expected total utility (willingness to pay) depends upon the consumers' perception of their consumption strategies as well as the penalty-fee level. If consumers believe that they do not need to track their consumption history, their perceived expected utility and other properties are the same as in the no-tracking condition. However, if consumers believe that they will need to track their consumption history, their perceived utility depends on the tracking cost k. As one would expect, as tracking cost k gets larger, consumers' perceived utility decreases. More interestingly, in this case, consumers' willingness to pay for the service is lower than $\frac{5}{8}$ when facing a high penalty fee. The reason is that consumers take the tracking cost into consideration when they construct their perceived expected utility before making the purchase decision, and that tracking

cost lowers their willingness to pay. That is to say, when the contract described in Proposition 1 is in effect, consumers will leave the market.

The FCA (2015) find that even when the text message registration is free for account users, very few (around 3%-8%) agree to sign up for it. This model enables me to determine a consumer's willingness to pay for the tracking option. A special case is that, given the optimal penalty fee derived in Proposition 1, consumers would prefer not to pay anything to check their consumption history as long as the tracking cost k is positive, a factor that explains the finding in FCA (2015) about the low enrollment rate. In general, consumers not planning to check their consumption history automatically implies that their willingness to pay for the tracking technology is less than 0. However, they would be willing to pay for the tracking option if they do plan to track their consumption history. In particular, consumers are willing to pay less if the tracking cost is significant or if consumers strongly believe that they have a much better memory than they actually do.

Given consumers' consumption strategies for each period as well as their tracking decision described earlier, We am able to compute the corresponding probability that they will choose to consume in both periods following $\Pr(d_1 = d_2 = 1) = \Pr(d_1 = 1) * \Pr(d_2 = 1 | d_1 = 1)$. If consumers choose to track their consumption in period two, they are able to avoid paying a penalty fee if ϕ is big due to the assumption that the maximum value they can get is 1. However, consumers might be willing to pay a penalty fee if they get higher value than the penalty fee, leading to a positive probability of consuming in both periods. On the other hand, if consumers do not track their consumption, the probability of incurring a penalty fee would be the same as was described in Section 3.2. Equation 5 summarizes the probability of being charged a penalty fee given consumers use the consumption tracking technology.

$$\Pr(d_1 = d_2 = 1) = \begin{cases} (1-\phi)(1-k-\phi+\frac{1}{2}\phi^2) & \phi \leq 1 \text{ and } k \leq k^*(\tilde{\alpha}, \phi) \\ (1-\phi+\frac{1}{2}(1-\tilde{\alpha}^2)\phi^2)(1-\phi) & \phi \leq 1 \text{ and } k^*(\tilde{\alpha}, \phi) < k \leq k^*(\alpha, \phi) \\ 0 & OW \end{cases} \quad (5)$$

Further investigation of $\Pr(d_1 = d_2 = 1)$ suggests that tracking cost levels affect the probability directly when consumers plan to check their consumption history and the penalty fee is less than 1 (first line in Equation 5). The reason is that when consumer plan to track their consumption, the probability of consuming in period one relies on k while period two decision only depends on the level of ϕ. As noted previously, as long as consumers track their consumption, they will track their consumption if the value they get is higher than the penalty fee, which is independent of k. As a result, when facing a low penalty fee, consumers will still choose to consume after tracking their consumption. Moreover, consumers are more likely to be charged a penalty fee as tracking costs decrease, because a lower tracking cost increases the probability of consuming in period one but does not affect their choice in period two. If the penalty fee is high, however, consumers will choose not to consume in period two and avoid paying a penalty fee because they tracked their consumption.

However, if consumers initially believe they do not need to, but then they do choose to track their consumption history, the probability of being charged a penalty fee does not depend on the tracking cost k or their forgetfulness α. Moreover, the probability of incurring a penalty fee weakly increases for consumers with smaller $\tilde{\alpha}$. As discussed earlier, if consumers do not check their consumption history, the probability of being charged a penalty fee increases the more forgetful or naive consumers are.

Given the service contract structure (p, ϕ), the firm's profit function can be represented as $\Pi = p + d_1 d_2 \phi$. As discussed earlier, the firm should set the sign-up price at the limit of consumers' willingness to pay in order to extract consumer surplus while ensuring that consumers will be willing to purchase the contract. We have discussed consumers' willingness to pay and the probability of incurring a penalty fee previously. It is important to note that if consumers successfully track their consumption, the sign-up price is the firm's only source of profit. In such a scenario, the firm benefits more from providing a contract with no penalty fee and a high sign-up price.

Further investigation of the firm's profit margins suggests that the firm generally earns a lower level of profit when consumers do, in fact, check their consumption history. More specifically, the firm's highest profit levels occur when forgetful consumers do not track their consump-

tion history, followed by instances when consumers plan not to track their consumption history but then decide to do so in period two, and then finally, in scenarios where consumers consider tracking their consumption before making any consumption decision. Notably, in the latter case, for the given level of penalty fee, the firm has to lower the sign-up price to ensure consumers make the purchase; otherwise, consumers will choose to leave the market instead.

Taken together, We can evaluate the effect of consumption tracking technology on consumers' behavior and on the firm's profit when the firm cannot adjust its contract. Overall, both the consumers' welfare and the firm's profit depend on how large the cost of tracking is. This model's findings generate the following conclusions: (i) if the tracking cost is high, the advancement of consumption tracking technology will not affect consumers behavior as well as firm's profit. (ii) if consumption tracking technology reduces tracking cost (k) such that $k^*(\tilde{\alpha}, \phi) \leq k \leq k^*(\alpha, \phi)$, consumers will not incur a penalty fee and will be better off, but the firm's profit comes from the sign-up price only and its profit will decrease; and (iii) if the tracking cost k is reduced to the extent that $k < k^*(\tilde{\alpha}, \phi)$, consumers have a lower willingness to pay for the contract and will then leave the market, and the firm's corresponding profit becomes zero.

To summarize, We have,

COROLLARY 1: *If the advancement of consumption tracking technology changes consumers anticipation or makes consumers track their consumption, the firm's profit declines and consumers surplus increases.*

The advancement of consumption tracking technology gives the firm an incentive to change its penalty fee such that consumers will choose not to track their consumption in period two. This is consistent with Sims (2003) rational inattention idea, whereby consumers choose to be inattentive after considering the cost of attention. By maximizing the firm's profit, We are able to devise the optimal penalty fee as follows:

$$\phi^* = \begin{cases} \frac{1}{2(1-\alpha)} & \alpha > \max(\frac{2}{3}, \frac{1+\tilde{\alpha}}{2}) \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{8} \\ \frac{1}{1-\tilde{\alpha}} & \frac{3\tilde{\alpha}^2-2\tilde{\alpha}+3}{4} < \alpha < \frac{1+\tilde{\alpha}}{2} \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ \frac{1-\sqrt{-2k+\alpha}}{1-\alpha} & \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \leq k < \frac{1}{2}\alpha - \frac{1}{8} \text{ and } \alpha > \frac{1+\tilde{\alpha}}{2} \\ & \text{or} \max\{k_1, \frac{1}{2}\alpha - \frac{1}{2}\alpha^2\} < k < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ 0 & OW \end{cases} \tag{6}$$

where k_1 makes the firm indifferent between charging $\phi = 0$ and $\phi = \frac{1-\sqrt{\alpha-2k}}{1-\alpha}$. The corresponding optimal sign-up price (p) can be calculated accordingly. The expressions for k_1 and p^* are shown in the Appendix.

How, then, shall we interpret these results? First, if the tracking cost is set so high that consumers will not choose to check their consumption history, the optimal penalty fee remains the same. Secondly, if the cost of using the tracking technology is low enough that consumers will expect to use it when they purchase the contract, the profit from any contract with a positive penalty fee is smaller than 1. However, if the firm changes its penalty fee to zero, the firm will charge a higher sign-up price to compensate for the loss from penalty fee revenues. In this situation, consumers choose to consume in both periods and all the firm's profit comes from an increased sign-up price, wherein forgetful consumers pay a higher sign-up price but they do not incur penalty fees. Overall, the increase in consumer surplus is greater than the decrease in the firm's profit, leading to an improvement in overall welfare.

More interestingly, if the tracking cost lies somewhere between the two cases such that consumers have a high probability of using it in period two, the firm will still charge a positive penalty fee. And that penalty fee will be set at the point where consumers are indifferent between tracking and not tracking, which is less than where the fee is set in the no-tracking-option case. This new optimal penalty fee increases with forgetfulness and cost of tracking. Although the new penalty fee makes consumers more likely to incur a penalty fee, the expected penalty fee paid is lower. The corresponding sign-up price depends on consumers' perception of how forgetful they are. It will stay the same for very naive consumers (such as the consumers of Region A in Figure 3) and increase

for relatively sophisticated consumers. As such, the firm's profit increases along with the tracking cost and the level of forgetfulness. Compared to the no-tracking-option case, consumers are better off, while the firm's profit decreases somewhat because it earns less revenue from penalty fees. Overall, this results suggests that the penalty fee may not be eliminated even when consumption tracking technology could make consumers aware of their period-one decisions. These findings are summarized in Proposition 2 below:

PROPOSITION 2: *The advancement of consumption tracking technology weakly decreases the penalty fee and weakly increases the sign-up price. Moreover, it improves consumer welfare and reduces firm's profit even if consumers do not use the tracking technology in equilibrium.*

Importantly, without price adjustment, the firm loses consumers who plan to track their consumption history and loses the penalty revenue from consumers who use the tracking technology. When the firm adjusts its contract according to Equation 6, the firm is able to serve all the consumers in the market, and naive forgetful consumers will still have a positive probability of incurring a penalty fee. Hence, the firm's profit increases with price adjustments, although it is still lower than the no-tracking-option case. To summarize, under the new plans, no consumers will be incentivized to track their consumption history as long as there is a positive tracking cost k associated with it. In the equilibrium many consumers benefit from the technology, not directly as they may not have the incentive to use the technology due to lower expected penalty, but benefit indirectly from the firm's response in price reduction. Our findings about the new optimal contract with tracking option derived from considering the firm's response are consistent with the findings in the survey by MoneyRate.com: the average annual cost for maintaining a checking account is increasing, but overdraft protection fees continue to decrease.[11].

To illustrate the optimal contract with tracking option, We draw different regions for the option contract in Figure 5 when $k = \frac{1}{4}$. Consumers from Regions A, B, and C have the same contract as is the case with the no-tracking-option shown in Figure 3, and they will not track their consumption history. We have discussed the rationale supporting the pricing scheme for these three regions in Section 3.2 and so will not repeat it here, but it should be noted that the size of

[11] https://www.heraldtribune.com/news/20190304/john-hielscher-state-taps-new-regulator-for-financial-services-providers

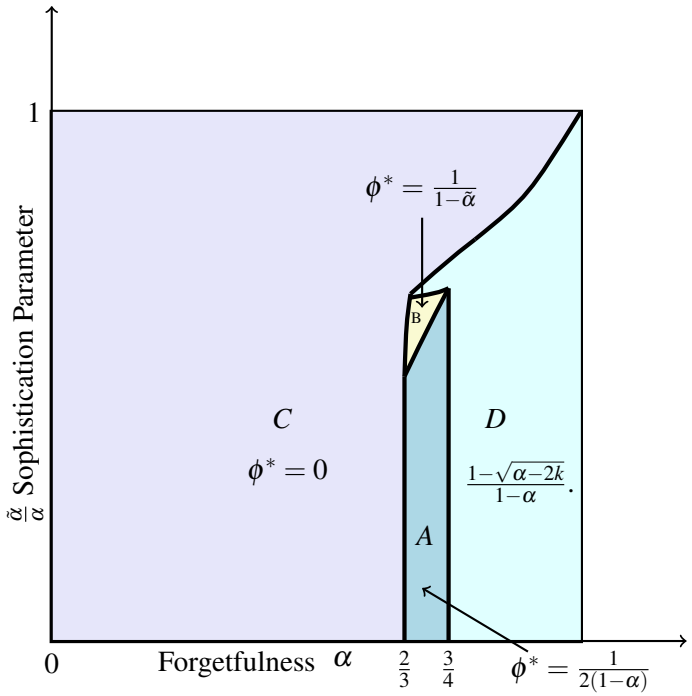

Figure 5: Regions in Optimal Penalty Fee with Tracking Option

Region C increases, while Regions A and B get smaller. With lower tracking costs, relatively sophisticated consumers are more likely to utilize the technology and track their consumption, and the firm will thus structure the contract with a zero penalty fees and a high sign-up price. Some consumers who were in Region B will then move to Region C when the tracking costs are lower, which also increases the size of Region C. The firm will also change the penalty fee for more forgetful consumers from Region A and for their more sophisticated consumers from Region B, because they would otherwise face a penalty fee. As a result, the size of regions A and B decreases.

Figure 3 showed three regions, but Figure 5 includes a new region Region D which includes two types of consumers, both of which had their penalty fee reduced to accommodate the advancement of the consumption tracking technology. Although consumers in Region D with the same α face the same penalty fee, their sign-up price differs. The consumer type that expects a zero probability of incurring a penalty fee's willingness to pay for the contract is $\frac{5}{8}$. The second consumer type in Region D, those who are relatively sophisticated about their forgetfulness, understand the positive probability of incurring a penalty fee and thus see more value in purchasing consumption

tracking technology. Overall, consumers in Region D face a lower penalty fee and benefit from having it available to them.

5 Heterogeneous Consumers

This section extends the principal model to focus on consumers with differing forgetfulness levels. Its main goal is to investigate the firm's optimal pricing strategy and to evaluate consumer welfare in a mixed market where consumption tracking technology is improved. As was the case previously, at each later period $t \in \{1,2\}$, consumers privately learn their value v_t, which follows Uniform distribution, $v_t \sim U[0,1]$. Allowing consumers from different regions to have varying levels of forgetfulness and perceptions provides a rich context to study consumer heterogeneity.

This section, then, focuses on two groups of consumers with the same level of perception $\tilde{\alpha}$ but with different levels of forgetfulness α. In particular, let there be $\sigma > 0$ consumers with α_L and the rest $1-\sigma > 0$ consumers with α_H. Moreover, $\alpha_L > \max\{\frac{2}{3}, \frac{1+\tilde{\alpha}}{2}\}$, which implies that consumers are from Region A in Figure 3. One reason for starting with this combination is that consumers from both groups expect 0 probability of incurring a penalty fee when deciding about the contract described in Proposition 1. In fact, their willingness to pay is the same since they share the same perception ($\tilde{\alpha}$). Therefore, the firm cannot conduct price discrimination for these two groups. Instead, it will provide one contract to serve both groups. In this section, We will start with the case of no consumption tracking options and then look at firm's pricing and consumer behavior with the advancement of consumption tracking technology. A complete analysis is shown in Appendix.

Before We move to the optimal contract in the mixed market, We first need to identify the optimal penalties for each group of consumers in the homogeneous market. One can deduce the optimal penalty fee for consumers who are not offered a tracking option relatively simply by applying Proposition 1. The optimal penalty charged for consumers without a tracking option is $\phi^* = \frac{1}{2(1-\alpha)}$. Correspondingly, the optimal penalty fee charged for the two groups of consumers will be denoted as ϕ_L^* and ϕ_H^* respectively.

Given the proportion of each group in the mixed market and their corresponding levels of forgetfulness, We are able to find the optimal contract for the two groups: if $\sigma > \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H-\alpha_L)}$, the optimal contract should be set as follows: $p_0^* = \frac{5}{8}$ and $\phi_0^* = \frac{1}{2(1-\sigma\alpha_L-(1-\sigma)\alpha_H)}$. Otherwise, the firm should focus on the more forgetful consumers and charge ϕ_H^*, which is what is charged in the homogeneous market. In this section, We will focus on the part when there are enough less forgetful consumers on the market ($\sigma > \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H-\alpha_L)}$). A complete analysis is provided in the Appendix.

If $\sigma > \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H-\alpha_L)}$, the optimal penalty fee (ϕ_0^*) falls between ϕ_H^* and ϕ_L^*. Compared to the market with homogeneous consumers, it seems that less forgetful consumers face a worse contract since they pay the same sign-up price and face a high penalty fee. In fact, both groups will pay less on penalty fees. The rationale is as follows: consumer welfare is lowest in the market with homogeneous consumers since the firm maximizes its profit without accommodating to the existence of other group of consumers. Any deviation from ϕ_L^* or ϕ_H^* will improve consumer welfare. Therefore, both groups of consumers are better off compared to the market with homogeneous consumers.

When the consumption tracking technology is improved, there is a possibility that consumers will choose to use it and thus avoid paying penalty fees. Therefore, given the advancement of consumption tracking technology, the optimal contract provided to those naive forgetful consumers can be derived by directly applying Equation 6:

$$(p^*, \phi^*) = \begin{cases} (\frac{5}{8}, \frac{1}{2(1-\alpha)}) & k \geq \frac{1}{2}\alpha - \frac{1}{8} \\ (\frac{5}{8}, \frac{1-\sqrt{\alpha-2k}}{1-\alpha}) & \frac{\alpha}{2} - \frac{(\alpha-\tilde{\alpha})^2}{2(1-\tilde{\alpha})^2} \leq k < \frac{\alpha}{2} - \frac{1}{8} \\ (1,0) & Otherwise \end{cases}$$

As explained in Section 4.2, the willingness to pay is the same for both groups of consumers who face a high penalty fee. As a result, the firm will continue to offer a single contract to them. We further assume that $k > \frac{\tilde{\alpha}}{2}$, implying the advancement of consumption tracking technology will not affect consumer's willingness to pay. With the assumption of α and k, the firm should set its

sign-up price to be $\frac{5}{8}$ to extract all their expected perceived surplus.

With the penalty fee ϕ_0^*, the thresholds for the two groups of consumers are denoted as $k^*(\alpha_H, \phi_0^*)$ and $k^*(\alpha_L, \phi_0^*)$ respectively. By comparing the two thresholds, less forgetful consumers are more likely to use the tracking technology ($k^*(\alpha_H, \phi_0^*) < k^*(\alpha_L, \phi_0^*)$) if $\sigma \geq \frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)}$. In this section, We focus on this case[12]. The reasoning is as follows: less forgetful consumers face a high penalty fee ϕ_0^*, and are more likely to recognize the value of consumption tracking technology, and thus more likely to use it. In addition, in terms of sophistication parameters, less forgetful consumers are more sophisticated than the more forgetful consumers. With lower tracking costs, the relative more sophisticated consumers are more likely to use it. In Appendix, We also provide analysis for $\sigma < \frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)}$.

To evaluate the effect of the consumption tracking technology on consumer behavior and firm's profit, We will first look at the case where the firm is not allowed to revise its contract. Depending on the level of tracking cost k, there are three possible scenario: 1. When the cost of using tracking technology is high enough that neither group of consumers will track their consumption ($k > k^*(\alpha_L, \phi_0^*)$), the advancement of consumption tracking technology will not affect consumers' behavior, or the firm's profit; 2. When k falls between the two thresholds, namely, $k^*(\alpha_H, \phi_0^*) < k < k^*(\alpha_L, \phi_0^*)$, those less forgetful consumers will choose to track their consumption while the more forgetful consumers will not. As a result, without price adjustment, less forgetful consumers will benefit from being able to avoid paying a penalty fee; and 3. When the cost associated with the consumption tracking technology is low enough that both groups of consumers will choose to track their consumption ($k < k^*(\alpha_H, \phi_0^*)$), the firm's only earns revenue from the sign-up price ($\frac{5}{8}$), which is lower than what it earns without the tracking technology. Both groups of consumers benefit from being able to avoid paying penalty fees with the tracking technology. In Appendix, We provide a complete analysis for the three cases described above.

The following analysis focuses on what happens when the less forgetful consumers track their consumption while the more forgetful group do not ($k^*(\alpha_H, \phi_0^*) < k < k^*(\alpha_L, \phi_0^*)$). In this

[12]Here, $\frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)} > \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H-\alpha_L)}$. It means that if $\sigma > \frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)}$, the optimal penalty fee in the case of no consumption tracking technology is ϕ_0^*

case, the firm loses the penalty fee revenue from less forgetful consumers. The reason for focusing on this case is as follows: In real life, we see the consumption tracking technology is getting popular. Some consumers are using it to track their consumption's, while the others are not. By investigating this case, the results can provide more insights to the market behavior, and also help us better understand the effect of consumption tracking technology. In this case, the inequality always holds: $\frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H} > \phi_0^* > \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$. If the firm is allowed to revise its contract, the firm will respond to the advancement of consumption tracking technology by either increasing or decreasing its penalty fee, depending on the relative size of each group.

If the proportion of the less forgetful consumers is small, the firm will only focus on the more forgetful consumers, and increases its penalty fee to exploit them while still ensuring that those more forgetful consumers will not track their consumption. In particular, if $k > \frac{1}{2}\alpha_H - \frac{1}{8}$, the firm can set the penalty fee as $\phi_H^* = \frac{1}{2(1-\alpha_H)}$ and more forgetful consumers will not choose to use the tracking technology. Otherwise, the firm has to accommodate the advancement of consumption tracking technology and sets the penalty fee as $\frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$, which is higher than ϕ_0^*, to prevent the more forgetful consumers from using it. In equilibrium, the penalty fee increases; the more forgetful consumers will not use the consumption tracking technology while the less forgetful consumers will use it. Although the firm will lose the penalty fee revenue from the less forgetful consumers, it earns a higher profit from those more forgetful consumers through increasing its penalty fee.

However, if the proportion of less forgetful consumers is large enough that the firm can not ignore this group, the firm will decrease its penalty fee such that the less forgetful consumers are indifferent from using the consumption tracking technology or not. In equilibrium, the penalty fee decreases and neither group of consumers will track their consumption. It is notable that $\frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$ is not necessary to be smaller than ϕ_L^*.

To summarize, we have the following proposition:

PROPOSITION 3: *If* $\sigma > \frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)}$*, the optimal penalty fee is* $\phi_0^* = \frac{1}{2(1-(\sigma\alpha_L+(1-\sigma)\alpha_H))}$ *without tracking option. When the cost associated with the tracking technology k is* $k^*(\alpha_L, \phi_0^*) < k < k^*(\alpha_H, \phi_0^*)$*,*

(i) the firm will increase the penalty fee to $\phi^* = \frac{1}{2(1-\alpha_H)}$ *if* $\sigma < \min(\sigma_2, 3\alpha_H - 2)$ *and* $k > \frac{1}{2}\alpha_H - \frac{1}{8}$*. In this case, the less forgetful consumers will use the tracking technology while more forgetful consumers will not;*

(ii) if $\sigma < \min(\sigma_1, -\frac{(-8k+\alpha_H-4\sqrt{-2k+\alpha_H}+3)}{8k-4\alpha_H+4\sqrt{-2k+\alpha_H}})$ *and* $k < \frac{1}{2}\alpha_H - \frac{1}{8}$*, the optimal penalty fee will be increased to* $\frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$*. In this case, the less forgetful consumers will use the tracking technology while more forgetful consumers will not;*

(iii) the firm will decrease the penalty fee to $\frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$ *if* $\frac{(-8k+2\alpha_L-4\alpha_H-4(1+\alpha_L-2\alpha_H)\sqrt{-2k+\alpha_L}+8k\alpha_H+3\alpha_L^2-4\alpha_L\alpha_H+3)}{4(\alpha_H-\alpha_L)(1-\sqrt{-2k+\alpha_L})^2} > \sigma > \max\{\sigma_2, \sigma_1\}$*. In this case, neither group of consumers will use the tracking technology;*

(iv) otherwise, the firm will set the penalty fee to be 0. In this case, neither group of consumers will use the tracking technology

Taking the firm's response into account, We are able to evaluate the effect of the advancement of consumption tracking technology on consumer welfare as well as on the firm's profit. If the firm *increases* the penalty fee, less forgetful consumers will use the tracking technology to avoid paying it. As a result, less forgetful consumers will benefit from the advancement of consumption tracking technology. The more forgetful consumers will not use the consumption tracking technology and face a higher penalty fee, one which is closer to ϕ_H^*, and they will be worse off as a result. The firm, on the other hand, earns a higher profit from more forgetful consumers, but it only gets the sign-up price from less forgetful consumers ; its profit is thus reduced by the advancement of consumption tracking technology.

However, if the proportion of less forgetful consumers is large, the firm *decreases* its penalty fee to prevent both consumer groups from using the tracking technology. Although the penalty fee decreases, both groups of consumers are more likely to incur penalty fees. Moreover, less forgetful consumers are expected to pay more in penalty fees if . In terms of the realized consumer surplus, less forgetful consumers have a lower realized consumer surplus if $-\frac{\alpha_L(\alpha_L^2-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1)}{(\alpha_L+1)^2(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)} + \frac{1}{2}\alpha_L - \frac{1}{8}(\frac{\alpha_L-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1}{\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1})^2 < k$. The more forgetful consumers will be better off. A detailed analysis is shown in the Appendix. This result suggests that With lower tracking costs, the seemingly sophisticated consumers could be worse off while the relative more naive consumers could

be better off. The reasoning is as follows: The advancement of consumption tracking technology leads the firm to reduce its penalty fee. The new penalty fee is closer to ϕ_L^*, and those consumers are worse off. The more forgetful consumers will benefit because the new optimal penalty fee ϕ^* is smaller than ϕ_H^* and the advancement of consumption tracking technology pushed the penalty fee further from ϕ_H^*.

The firm's profit decreases as the proportion of less forgetful consumers increases, or as tracking cost decreases due to the techonogical advancement. To summarize, We have the following proposition:

PROPOSITION 4: *If the firm decreases its penalty fee, both groups of consumers are more likely to incur a penalty fee. Moreover, despite neither groups of consumers use the consumption tracking technology, less forgetful consumers are worse off by paying more on penalty fees, while the more forgetful consumers are better off if* $-\frac{\alpha_L(\alpha_L^2-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1)}{(\alpha_L+1)^2(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)}+\frac{1}{2}\alpha_L-\frac{1}{8}(\frac{\alpha_L-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1}{\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1})^2 <$ *k. However, if the firm increases its penalty fee and only focuses on the more forgetful consumers, the less forgetful consumers will track their consumption and become better off while more forgetful consumers will not track and become worse off.*

The numerical example below illustrates this mechanism:

Numerical Example: Values are uniformly distributed on $[0,1]$, there are two groups of consumers, $\frac{1}{2}$ of them are with level of inattention $\alpha_L = .7$ and the rest are more forgetful with $\alpha_H = .9$. Both types of consumers are fully naive ($\tilde{\alpha} = 0$) and share the same tracking cost k. Moreover, the tracking cost associated with the consumption tracking technology can is .25.

Table 1: Equilibrium Outcome with Heterogeneous Consumers

	No consumption tracking option		$k = .23$	
	$\alpha_L = .7$	$\alpha_H = .9$	$\alpha_L = .7$	$\alpha_H = .9$
sign-up price	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$
Penalty fee	2.5	2.5	1.84	1.84
Probability of Penalty fee	.125	.375	.224	.41
Total penalty	.31	. 94	.41	. 75
Profit	1.25		1.21	
CS	-.2	-.7	-.24	-.5

Table 1 illustrates that the sign-up price does not change as the firm reduces the penalty fee to respond to the existence of consumption tracking technology. Although both groups are more likely to incur a penalty fee, the expected fee paid by less forgetful consumers increases. Moreover, the increase in total penalty fees paid by less forgetful consumers increases by 32%, while the reduction in total penalty fees paid by the more forgetful consumers is only 20%. In other words, when facing a decrease in tracking cost, the less forgetful consumers suffer, while the more forgetful consumers benefit. From the firm's point of view, the total profit decreases from 1.21 to 1.25, which is a decrease of approximately 3%. Overall, the impact of decreased tracking costs on the firm's profit is very limited, and less forgetful consumers could be worse off, which is contradictory to the finding in the market with homogeneous consumers.

6 Policy Analysis

Previous sections outlined the implications that consumption tracking software had for firm profits, consumer surplus (CS), and total welfare. This section focuses on potential policies that aim to reduce the cost of tracking one's consumption history and provide consumers a convenient way to be informed about their previous activities. Overdraft fees comprise a large portion of what banks refer to as "non-interest income." They contribute approximately one-third of banks' profits from current accounts, and regulatory authorities have enacted different policies in attempts to lower this. For example, the FCA in the UK initiated giving consumers the option to receive text message alerts regarding overdrafts in 2012, and then in 2018, auto-enrolled all consumers to receive overdraft-related text message alerts. In terms of the effectiveness of these initiatives, the sign-up rate for text message alert is between 3% and 8%, suggesting that the policy's impact is limited. The auto-enrollment program significantly reduced the overdraft payment overall FCA (2018). These effects were typically evaluated when the banks did not change their contracts to accommodate the policy change. The model presented in this research captures the firm's response, and can provide a perspective to fully understand and evaluate these policies.

This paper suggests that the utility of policies that aim to reduce the cost of tracking one's

consumption depends on both how significant the reduction is. We examine this by first looking at the scenario where the firm cannot change its contracts, and then by taking the firm's price adjustment into account. Without price adjustment, three results are possible based on the optimal contract shown in Equation 6.

In the first instance, a small reduction will not affect the behavior of consumers who have a relatively high tracking cost and face a penalty fee independent of k, and they will continue to not track their consumption history. However, if the reduction in the cost of tracking is larger, such that $k^*(\phi^*, \tilde{\alpha}) < k < k^*(\phi^*, \alpha)$, consumers will choose to track their consumption history and avoid paying penalty fees under the current plan. In that case, those consumers benefit while the firm profits less. Furthermore, if the reduction is large enough, such that $k < k^*(\phi^*, \tilde{\alpha})$, it will backfire since the reduction reduces consumers' willingness to pay and leads consumers to not purchase the contract. If so, both the firm and consumers get 0 surplus. In the second case, if consumers face a penalty fee depending on k, a small reduction in k will lead consumers to track their consumption history, thus increasing consumer welfare and decreasing the firm's profit since consumers will not pay a penalty fee. Similar to the discussion earlier, a larger reduction in tracking cost might drive consumers out of the market. As a result, it reduces the firm's profits. Third, if consumers face no penalty fees at all, they will never choose to check their consumption history and decreasing tracking costs will have no impact at all.

In a market with consumers with homogeneous levels of perception and forgetfulness, if the firm experiences a decrease in profits as a result of consumers' tracking behavior, the firm will strategically reduce its penalty fee to prevent consumers from tracking their consumption history. Therefore, in instances when a policy decreases the tracking cost using a price adjustment, the penalty fee will also decrease, while the sign-up might either increase or not change depending on the sophistication parameter. Although the resulting consumer surplus is still negative, it improves, as does total welfare, while the firm's profit decreases. As a result, implementing a decreased tracking cost could benefit both consumers and society in a homogeneous market. However, in a mixed market of consumers with heterogeneous levels of forgetfulness, if the proportion of less forgetful consumers is large, the firm will reduce its penalty fee to maximize its profit, resulting

in less forgetful consumers having to pay more penalty fees. On the other hand, if the proportion of less forgetful consumers is small, the firm will instead increase the penalty fee to exploit more forgetful consumers, As a result, the more forgetful consumers will be worse off while the less forgetful consumers will benefit by using the tracking technology. At the same time, firm's profit does not decrease much.

To summarize, then, without price adjustment, policies designed to reduce tracking costs could protect consumers from paying penalty fees, but might also make some consumers leave the market and not purchase the tracking services. With price adjustment, the reduction in tracking cost could benefit consumers and social welfare, but not as much as would be the case without price adjustment. The improvement might be inverted in a market with consumers with heterogeneous levels of forgetfulness, wherein some consumers could be worse off and the firm's profit does not change significantly. The overall effect of policies that reduce tracking costs in the mixed market can be ambiguous.

7 Conclusion

This research studies the marketing implications of the effect of consumption tracking technology when consumers have imperfect memory from a theoretical point of view. It extends previous literature on consumer inattention by incorporating consumers who can be partially forgetful about their past behavior and may not fully anticipate their forgetfulness. Moreover, the model incorporates the decision-making process of using the consumption tracking technology. In a monopolistic setting, We demonstrate that the advancement of consumption tracking technology will prompt consumers who plan and track their consumption to leave the market if the firm does not adjust its contract, as they take the cost of tracking their consumption into account and are thus less willing to pay for the service. When the firm is allowed to respond, it will reduce its penalty fee to disincentivize consumers from tracking their consumption. As a result, having the consumption tracking technology can be beneficial to both consumers and to society even if consumers do not use it. However, having the consumption tracking technology tends to detract from the wellbe-

ing of some type of consumers. With respect to the firm's profit, having consumption tracking technology tends to decrease the firm's profit overall, but not as much as when consumers are less forgetful or more aware of their forgetfulness.

The model also deepens our understanding of policies that focus on reducing tracking costs. If that cost can be driven to a significantly low level, then consumers could benefit even though the firm is likely to increase its sign-up price; realized consumer surplus and total welfare will both increase overall. Otherwise, the reduction in tracking cost will either have no effect or possibly even a negative effect for some type of consumers with heterogeneous levels of forgetfulness.

Future research might consider extending the model in several directions to seek additional insights into the effects of consumption tracking technology. First, including partially naive and partially forgetful consumers would allow the study of a mixed market with different types of consumers; this model only examined one combination of consumers who shared similar beliefs about their forgetfulness but had different levels of forgetfulness. One could extend the model to include consumers who share the same level of forgetfulness but have differing anticipations about how forgetful they are. A simple combination is to focus on fully naive forgetful consumers and sophisticated forgetful consumers. In that case, the firm will conduct price discrimination to maximize its profit, and having one group of consumers use the consumption tracking technology could result in equilibrium. Secondly, this paper addresses consumption tracking technology provided by a third party. However, in real life, we observe that banks and cellular phone companies often provide their own software applications to provide consumers access to their account balances. However, the quality of these applications tends to be inferior to those provided by third parties. One could also then extend the model by examining the firm's decision-making process regarding releasing its own the consumption tracking tool. We suspect the release of a firm's own version of a consumption tracking tool would ameliorate the effects of improving consumption tracking technology. It would also give the firm incentives/motivations to provide consumers with its own inferior version of a consumption tracking tool.

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A Notation

v_t	Value consumers get at period $t \in \{1,2\}$. It creates purchase probability
d_t	Consumption decision in period $t \in \{1,2\}$
$\tilde{\alpha}$	Ex-ante perception about forgetfulness
α	Ex-post realized forgetfulness
μ	$\mu = \frac{\tilde{\alpha}}{\alpha}$, sophistication parameter. It is used to measure how sophisticated consumers.
ϕ	Penalty fee
p	Sign-up price for services
Π	Profit for the firm
k	Tracking cost
σ	Proportion of less forgetful consumers
α_L	The level of forgetfulness for less forgetful consumers
α_H	The level of forgetfulness for more forgetful consumers

B Detailed Proofs

LEMMA 1: *For consumers with perfect memory, it is optimal for the firm to charge a high sign-up price ($p = 1$) and not to charge a penalty fee ($\phi = 0$). Moreover, this contract maximizes total welfare.*

Proof: When consumers have perfect memory ($\alpha = \tilde{\alpha} = 0$), their consumption strategies ($\underline{v_1}$ and $\underline{v_2}$) have been discussed in Section 3.1. Given the distribution of v_t, consumers perceived expected utility (willingness to pay) can be calculated through the following equation:

$$E(\tilde{U}) = (1-\underline{v_1})(\frac{1+\underline{v_1}}{2} + (1-\underline{v_2})(\frac{1+\underline{v_2}}{2} - \phi)) + \frac{\underline{v_1}}{2}$$

. Therefore, consumer's willingness to pay can be written as follows:

$$p = \begin{cases} \frac{5}{8}, & \phi > 1 \\ 1 - \phi + \phi^2 - \frac{1}{2}\phi^3 + \frac{1}{8}\phi^4, & \phi \leq 1 \end{cases}.$$

If the penalty fee is higher than the maximum value consumers can get, consumers with perfect memory will only consume once over two periods. In period one, consumers will choose to consume only if the value is greater than $\frac{1}{2}$, making the perceived expected utility as $E(\tilde{U}) = \frac{5}{8}$. However, if the penalty fee is small ($\phi \leq 1$), when consumers plan their consumption strategies, they will take the probability of

incurring a penalty fee into consideration, and the only way that they get charged a penalty fee is when they know it will happen, but choose to consume since they get higher value from it. That is to say, conditional on consumers consumed in period one, consumers will only choose to consume if the value they get from period two is higher than the penalty fee. According to the formula for perceived expected utility, we get $E(\tilde{U}) = 1 - \phi + \phi^2 - \frac{1}{2}\phi^3 + \frac{1}{8}\phi^4$.

The firm's profit comes from sign-up price ans revenue from penalty fee. As a monopoly firm, the sign-up price will be set at consumer's willingness to pay, which is consumer's expected perceived utility. Given those consumers consumption strategies as well as their willingness to pay, the firm's profit can be obtained through

$$\Pi = p + \Pr(d_1 = d_2 = 1)\phi$$

And it can be expressed as follows

$$\Pi = \begin{cases} \frac{5}{8}, & \phi > 1 \\ 1 - \phi^2\big(1 - \phi + \frac{3\phi^2}{8}\big), & \phi \leq 1 \end{cases}.$$

The firm will provide the contract that maximizes its profit. According to the profit function shown above, there are two cases:

Case 1: $\phi > 1$

$\Pi = \frac{5}{8}$

Case 2: $\phi \leq 1$

$\Pi = 1 - \phi^2((1-\phi) + \frac{1}{8}\phi^2)$

By taking derivatives of the profit function, We get $\frac{d\Pi}{d\phi} = -\frac{1}{2}\phi(\phi^2 - 6\phi + 4)$. The profit functions will have its local maximum point at $\phi = 0$, and the corresponding profit is $\Pi = 1$. While for the extreme value $\phi = 1$, the level of profit is $\Pi = \frac{7}{8}$. Therefore, it is optimal for the firm to provide a contract with 0 penalty fee and high sign-up price.

Comparing case 2 with case 1, firm has a higher profit when setting a high sign-up price and zero penalty fee. Therefore, the optimal penalty fee is $\phi^* = 0$, and the corresponding sign-up price is $p^* = 1$. In equilibrium, $\Pi^* = 1$ and consumers will consume in both periods. Moreover, the total welfare is maximized as well.

PROPOSITION 1: *It is optimal for the firm to charge (i)* $p^* = \frac{5}{8}$ *and* $\phi^* = \frac{1}{2(1-\alpha)}$ *if* $\alpha \geq \max(\frac{1+\tilde{\alpha}}{2}, \frac{2}{3})$*; (ii)* $p^* = \frac{5}{8}$ *and* $\phi^* = \frac{1}{1-\tilde{\alpha}}$ *if* $\frac{3\tilde{\alpha}^2 - 2\tilde{\alpha} + 3}{4} < \alpha \leq \frac{1+\tilde{\alpha}}{2}$*; (iii) otherwise,* $p^* = 1$ *and* $\phi^* = 0$*.*

Proof: Consumer's consumption strategies and their expressions for perceived expected utility (willingness to pay), as well as their probability of incurring a penalty fee have been shown in Equation (2) to (4). Here, We will not repeat the process. The firm's profit is

$$\Pi = p + \Pr(d_1 = d_2 = 1) * \phi$$

where the expressions for willingness to pay and probability of incurring a penalty fee depend on the ϕ. As a result, firm's profit is a function of penalty fee as well.

$$\Pi = \begin{cases} \frac{5}{8}, & \alpha < 1 - \frac{1}{\phi} \\ \frac{5}{8} + \frac{1}{2}(1-(1-\alpha)\phi)\phi, & \tilde{\alpha} \leq 1 - \frac{1}{\phi} \leq \alpha \\ 1 - \phi^2\big((1-\alpha)(1-\phi) + \frac{1}{8}(4\tilde{\alpha}^2 + (1-\tilde{\alpha}^2)(3+\tilde{\alpha}^2-4\alpha)\phi^2)\big), & 1 - \frac{1}{\phi} < \tilde{\alpha} \leq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \\ \frac{1}{2}, & \tilde{\alpha} > \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \end{cases}.$$

To find the firm's profit, We will look into each case and then find the optimal contract for each case, them compare different cases.

Case 1: $\alpha \leq 1 - \frac{1}{\phi}$

$\Pi = \frac{5}{8}$

Case 2: $\tilde{\alpha} \leq 1 - \frac{1}{\phi} \leq \alpha$

The condition can also be re-written as $\frac{1}{1-\tilde{\alpha}} \leq \phi \leq \frac{1}{1-\alpha}$

$\max_\phi \Pi = \frac{5}{8} + \frac{1}{2}(1-(1-\alpha)\phi)\phi$

FOC:

$\frac{d\Pi}{d\phi} = \alpha\phi - \phi + \frac{1}{2}$=0

The optimal penalty fee should be set at $\phi = \frac{1}{2(1-\alpha)}$. However, the condition $\tilde{\alpha} \leq 1 - \frac{1}{\phi} \leq \alpha$ needs to be satisfied. And, it requires $\alpha \geq \frac{\tilde{\alpha}+1}{2}$. Therefore, the optimal penalty fee is $\phi = \frac{1}{2(1-\alpha)}$ if $\alpha \geq \frac{\tilde{\alpha}+1}{2}$. If $\alpha < \frac{\tilde{\alpha}+1}{2}$, it implies that $\frac{1}{2(1-\alpha)} < \frac{1}{1-\tilde{\alpha}}$. Given the condition constraint, the optimal penalty fee would be set at $\phi = \frac{1}{1-\tilde{\alpha}}$. Therefore, We have the following conclusions:

(i) If $\alpha \geq \frac{\tilde{\alpha}+1}{2}$, the optimal penalty fee is $\phi = \frac{1}{2(1-\alpha)}$ and the corresponding sign-up price is $\frac{5}{8}$. In this case, the level of profit is $\Pi = \frac{5\alpha-6}{8(\alpha-1)}$.

(ii) If $\alpha < \frac{\tilde{\alpha}+1}{2}$, the optimal penalty fee is $\phi = \frac{1}{1-\tilde{\alpha}}$ and the corresponding sign-up price is $\frac{5}{8}$. In this case, the level of profit is $\Pi = \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$

Case 3: $1 - \frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$

$\Pi = (\frac{1}{2}\alpha - \frac{1}{2}\alpha\tilde{\alpha}^2 + \frac{1}{8}\tilde{\alpha}^4 - \frac{1}{8})\phi^4 + \phi^3 - \phi^2 - \frac{1}{2}\tilde{\alpha}^2\phi^2 + \alpha\phi^2 - \alpha\phi^3 + 1$

Instead of taking derivatives of the profit function with respect to ϕ, We focus on $\tilde{\alpha}$ first.

By taking derivative of the profit function with respect to $\tilde{\alpha}$, We get $\frac{d\Pi}{d\tilde{\alpha}} = -\frac{1}{2}\tilde{\alpha}\phi^2((2\alpha - \tilde{\alpha}^2)\phi^2 + 2)$

The local maximum happens when $\tilde{\alpha} = 0$. To satisfy the condition $1 - \frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$, the penalty fee should be $\phi \leq 1$. In this case, the firm's profit is $\Pi = (\frac{1}{2}\alpha - \frac{1}{8})\phi^4 + \phi^3 - \phi^2 + \alpha\phi^2 - \alpha\phi^3 + 1$. A further investigating on this profit function suggests that the firm is better off by charging $\phi = 0$ and $p = 1$, and the level of profit is $\Pi = 1$

At extreme values: first, if $\tilde{\alpha} = 1 - \frac{1}{\phi}$, the level of profit is $\Pi = \frac{1}{2}\phi - \frac{3}{4}\phi^2 + \frac{1}{2}\phi^3 + \frac{1}{2}\alpha\phi^2 + \frac{4}{8}$ Second, if $\tilde{\alpha} = \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$, the level of profit is $\Pi = \frac{1}{2}\phi^3 - \frac{1}{2}\phi^2 + \frac{1}{2}$. By comparing the two extreme values, We find that the profit when $\tilde{\alpha} = 1 - \frac{1}{\phi}$ is always higher: $\frac{1}{2}\phi - \frac{3}{4}\phi^2 + \frac{1}{2}\phi^3 + \frac{1}{2}\alpha\phi^2 + \frac{4}{8} > \frac{1}{2}\phi^3 - \frac{1}{2}\phi^2 + \frac{1}{2}$. It is notable that $\tilde{\alpha} = 1 - \frac{1}{\phi}$ can also be written as $\phi = \frac{1}{1-\tilde{\alpha}}$ and the level of profit in terms of $\tilde{\alpha}$ can be written as $\Pi = \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$

In addition, compared to the local maximum point, the firm still prefers to provide a positive penalty fee ($\phi = \frac{1}{1-\tilde{\alpha}}$) if the profit earned is higher than 1. Hence, we have the following conclusions:

(i) If $\alpha \geq \frac{3}{4}\tilde{\alpha}^2 - \frac{1}{2}\tilde{\alpha} + \frac{3}{4}$, the optimal penalty fee is $\phi = \frac{1}{1-\tilde{\alpha}}$ and the level of profit is $\Pi = \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$

(ii) If $\alpha < \frac{3}{4}\tilde{\alpha}^2 - \frac{1}{2}\tilde{\alpha} + \frac{3}{4}$, the optimal penalty fee is $\phi = 0$ and the level of profit is $\Pi = 1$

Case 4: $\tilde{\alpha} \geq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$

$\Pi = \frac{1}{2}$

With the optimal contract derived in each cases, We am able to compare different cases and find out the optimal contract that maximize firm's profit. In particular, We have

(i) $\frac{5\alpha-6}{8(\alpha-1)} > 1$ if $\alpha > \frac{2}{3}$

(ii) $\frac{5\alpha-6}{8(\alpha-1)} > \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$ if $\alpha > \frac{1+\tilde{\alpha}}{2}$

(iii) $\frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2} > 1$ if $\alpha > \frac{3}{4}\tilde{\alpha}^2 - \frac{1}{2}\tilde{\alpha} + \frac{3}{4}$

Therefore, it is optimal for the firm to charge (i) $p^* = \frac{5}{8}$ and $\phi^* = \frac{1}{2(1-\alpha)}$ if $\alpha \geq \max(\frac{1+\tilde{\alpha}}{2}, \frac{2}{3})$; (ii) $p^* = \frac{5}{8}$ and $\phi^* = \frac{1}{1-\tilde{\alpha}}$ if $\frac{3\tilde{\alpha}^2-2\tilde{\alpha}+3}{4} < \alpha \leq \frac{1+\tilde{\alpha}}{2}$; (iii) otherwise $p^* = 1$ and $\phi^* = 0$.

LEMMA 2: *Forgetful consumers will choose to check their consumption history if and only if* $k \leq k^*(\alpha, \phi)$

Proof: Consumers compare the two expected utility for period two before making the tracking decision.

First, if consumers choose not to track their consumption, at period two, they realize there is α probability they remember wrong. If the penalty fee is high enough such that $\alpha < 1 - \frac{1}{\phi}$, they will choose not to consume in period two and get 0 utility. That is, $E_b(u_2|d_1 = 1, b = 0) = 0$. On the other hand, if the penalty fee is lower such that $\alpha > 1 - \frac{1}{\phi}$, the threshold of consuming in period two is $(1-\alpha)\phi$ and the corresponding expected utility from period two is $E_b(u_2|d_1 = 1, b = 0) = \frac{(1-((1-\alpha)\phi))^2}{2}$.

$$E_b(u_2|d_1 = 1, b = 0) \begin{cases} 0, & \alpha < 1 - \frac{1}{\phi} \\ \frac{1}{2}(1 - ((1-\alpha)\phi))^2, & \alpha \geq 1 - \frac{1}{\phi} \end{cases}$$

Before tracking their consumption, they still believe that they have α probability to make a mistake. If they find that they have consumed before after paying k, facing a high penalty fee, they will not consume and get 0 utility. Then overall, their expected utility is $E_b(u_2|d_1 = 1, b = 1) = \frac{1}{2}\alpha - k$. Similar argument applies to the case when penalty fee is smaller than 1 , and in that case, consumers might still choose to consume even if they find that have consumed before and the expected utility from period two is $E_b(u_2|d_1 = 1, b = 1) = (1-\alpha)((1-\phi)(\frac{1+\phi}{2} - \phi)) + \frac{1}{2}\alpha - k$. Therefore, if consumers decide to track their consumption, their expected utility from period two before tracking can be written as follows:

$$E_b(u_2|d_1 = 1, b = 1) \begin{cases} \frac{1}{2}(1-\alpha)(1-\phi)^2 + \frac{\alpha}{2} - k, & \phi \leq 1 \\ \frac{1}{2}\alpha - k, & \phi > 1 \end{cases}$$

Hence, consumers will choose to track their consumption if the expected utility for period two is higher in the tracking case. And the threshold k^* that makes consumers indifferent from tracking or not can be derived through $E_b(u_2|d_1 = 1, b = 1) - E_b(u_2|d_1 = 1, b = 0) = 0$

Thus, we have,

$$k^*(\alpha,\phi)=\begin{cases}\frac{1}{2}\alpha, & \phi>\frac{1}{1-\alpha}\\ \frac{1}{2}\alpha-\frac{1}{2}(1-(1-\alpha)\phi)^2, & 1<\phi\leq\frac{1}{1-\alpha}\\ \frac{1}{2}\alpha(1-\alpha)\phi^2, & \phi\leq 1\end{cases}$$

COROLLARY 1: *Without price adjustment, if the advancement of consumption tracking technology changes consumer anticipation or makes consumers track their consumption, the firm is worse off and consumers surplus increases.*

Proof: Facing the contract described in Equation 5, if the advancement of consumption tracking technology changes consumers anticipation, according to Equation 8, the probability they consume in period one is $\frac{1}{2}-k$ when facing a penalty fee $\phi>1$. And in period two, when they track their consumption and find that they have consumed in period one, they will not choose to consume in period two; otherwise, they will consume in period two. Therefore, their willingness to pay become $p=\frac{1}{2}k^2-\frac{1}{2}k+\frac{5}{8}$, which is smaller than the sign-up price ($\frac{5}{8}$). As a result, consumers will leave the market and get value from their outside option (normalized to 0) and firm gets 0 profit. While in the case of no tracking option, when facing a contract with positive penalty fees, consumers' realized utility is negative and firm earns a higher than 1 profit ($\Pi>1$).

If the advancement of consumption tracking technology does not affect consumer's perception (willingness to pay), but changes consumers behavior such that consumers will use it to track their consumption. As discussed above, the optimal penalty fees described in equation 5 are either 0 or greater than 1. When facing a contract with 0 penalty fee, consumers will never choose to track their consumption. Therefore, when facing a contract with high penalty fee ($\phi>1$), tracking one's consumption will always help consumers avoid paying penalty fee. Therefore, the firm only gets revenues from sign-up price and earns 0 profit from penalty fee. To summarize, when consumers use the tracking technology, without pricing adjustment, they are able to avoid paying penalty fee, and get 0 net realized expected utility, and the firm loses profit from penalty fees. While in the case of no tracking option, when facing a contract with positive penalty fees, consumers' realized utility is negative and firm earns a higher than 1 profit ($\Pi>1$).

To conclude, without price adjustment, if the advancement of consumption tracking technology changes consumer anticipation or makes consumers track their consumption, the firm is worse off and consumers surplus increases.

PROPOSITION 2: *The advancement of consumption tracking technology weakly decreases the penalty fee and weakly increases the sign-up price. Moreover, it strictly improves consumer welfare and weakly reduces firm's profit even if consumers do not use the tracking technology in equilibrium*

Proof: The proof follows directly from Equation 9 and consumer's willingness to pay which is shown below:

$$p=\begin{cases}\frac{5}{8}, & 1-\frac{1}{\phi}\geq\tilde{\alpha}\text{ and }k>k^*(\tilde{\alpha},\phi)\\ \frac{1}{2}k^2-\frac{1}{2}k+\frac{5}{8}, & \phi>1\text{ and }k\leq k^*(\tilde{\alpha},\phi))\\ 1-\phi+(1-\frac{\tilde{\alpha}^2}{2})\phi^2-\frac{1-\tilde{\alpha}^2}{2}\phi^3+\frac{(1-\tilde{\alpha}^2)^2}{8}\phi^4, & 1-\frac{1}{\phi}<\tilde{\alpha}\leq\sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}\text{ and }k>k^*(\tilde{\alpha},\phi)\\ \frac{1}{2}k^2-\frac{1}{2}k\phi^2+k\phi-k+\frac{1}{8}\phi^4-\frac{1}{2}\phi^3+\phi^2-\phi+1\ , & \phi\leq 1\text{and }k\leq k^*(\tilde{\alpha},\phi)\\ \frac{1}{2}, & OW\end{cases}$$

And the firm's profit comes from the sign up price and the revenue from penalty fees.

$$\Pi = p + \Pr(d_1 = d_2 = 1) * \phi$$

where the expressions for willingness to pay and probability of incurring a penalty fee depend on the ϕ. It can be expressed as follows:

$$\Pi = \begin{cases} \frac{5}{8}, & 1-\frac{1}{\phi} > \alpha \text{ and } k \geq k^*(\tilde{\alpha},\phi) \\ & \text{or } \tilde{\alpha} \leq 1-\frac{1}{\phi} < \alpha \text{ and } k^*(\tilde{\alpha},\phi) \leq k < k^*(\alpha,\phi) \\ \frac{1}{2}k^2 - \frac{1}{2}k + \frac{5}{8}, & \phi > 1 \text{and } k < k^*(\tilde{\alpha},\phi) \\ 1-\phi+(1-\frac{\tilde{\alpha}^2}{2})\phi^2 - \frac{1-\tilde{\alpha}^2}{2}\phi^3 + \frac{(1-\tilde{\alpha}^2)^2}{8}\phi^4, & 1-\frac{1}{\phi} < \tilde{\alpha} \leq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \text{and } \phi > 1 \\ & \text{and } \ k^*(\tilde{\alpha},\phi) \leq k < k^*(\alpha,\phi) \\ \frac{1}{2}k^2 + \frac{1}{2}k\phi^2 - k - \frac{3}{8}\phi^4 + \phi^3 - \phi^2 + 1 \ , & \phi \leq 1 \text{ and } \ k < k^*(\tilde{\alpha},\phi) \\ \frac{\tilde{\alpha}^4\phi^4}{8} + \frac{\tilde{\alpha}^2\phi^4}{4} - \frac{\tilde{\alpha}^2\phi^2}{2} - \frac{3\phi^4}{8} + \phi^3 - \phi^2 + 1, & \phi \leq 1 \text{and } k^*(\tilde{\alpha},\phi) \leq k < k^*(\alpha,\phi) \\ \frac{5}{8} + \frac{1-(1-\alpha)\phi}{2}\phi, & \tilde{\alpha} \leq 1-\frac{1}{\phi} \leq \alpha \text{ and } k \geq k^*(\alpha,\phi) \\ 1-\phi^2((1-\alpha)(1-\phi) + \frac{1}{8}(4\tilde{\alpha}^2 + (1-\tilde{\alpha}^2)(3+\tilde{\alpha}^2-4\alpha)\phi^2))), & 1-\frac{1}{\phi} < \tilde{\alpha} \leq \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}} \text{and } k \geq k^*(\alpha,\phi) \\ \frac{1}{2}, & OW \end{cases}$$

By maximizing the firm's profit, We are able to obtain the optimal contract with the advancement of consumption tracking technology. For consumers who plan to use the tracking technology, the firm will provide a contract with zero penalty fee and high sign-up price. For consumers who plan not to track their consumption but will do so, the firm will intentionally reduce the penalty fee to disincentivize them from using it. For those consumers who do not track their consumption, there is no need for the firm to change its contract.

Case 1A: $1-\frac{1}{\phi} > \alpha$ and $k > k^*(\tilde{\alpha},\phi)$

$\Pi = \frac{5}{8}$

Case 1B: $\tilde{\alpha} < 1-\frac{1}{\phi} < \alpha$ and $k^*(\tilde{\alpha},\phi) \leq k < k^*(\alpha,\phi)$

$\Pi = \frac{5}{8}$

Case 2: $\phi > 1$and $k < k^*(\tilde{\alpha},\phi)$

$\Pi = \frac{1}{2}k(k-1) + \frac{5}{8} < 1$

Case 3: $1-\frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$and $\phi > 1$ and $k^*(\tilde{\alpha},\phi) \leq k \leq k^*(\alpha,\phi)$

$\Pi = 1-\phi+(1-\frac{\tilde{\alpha}^2}{2})\phi^2 - \frac{1-\tilde{\alpha}^2}{2}\phi^3 + \frac{(1-\tilde{\alpha}^2)^2}{8}\phi^4$

In this case, the advancement of consumption tracking technology will not change consumer's perception, thus their willingness to pay. However, consumers will choose to track their consumption and will not pay a penalty fee. As a result, the firm only receives revenue from sign-up price. Since the penalty fee is greater than 1, consumers will not plan to consume in both periods and pay a penalty fee. It makes their willingness to pay strictly lower than 1. Therefore, the level of profit, which is the sign-up price is lower than 1.

Case 4: $\phi \leq 1$ and $k < k^*(\tilde{\alpha},\phi)$

$\Pi = \frac{1}{2}k^2 + \frac{1}{2}k\phi^2 - k - \frac{3}{8}\phi^4 + \phi^3 - \phi^2 + 1$

In this case, the advancement of consumption tracking technology will not only make consumers to track their consumption, but also changes consumer's perception, thus their willingness to pay. As discussed before, by taking the tracking cost into account, consumers' willingness to pay is lower. Although consumers might still choose to consume in both periods after checking, the overall profit earned by the firm is smaller than 1. Hence, the firm would rather to provide a contract with 0 penalty fee.

Case 5: $\phi \leq 1$ and $k^*(\tilde{\alpha}, \phi) \leq k < k^*(\alpha, \phi)$

$\Pi = (\frac{1}{8}\tilde{\alpha}^4 + \frac{1}{4}\tilde{\alpha}^2 - \frac{3}{8})\phi^4 - \frac{1}{2}\tilde{\alpha}^2\phi^2 + \phi^3 - \phi^2 + 1$

In this case, consumers face a lower penalty fee and will track their consumption, but they might still choose to consume in both periods after checking. As shown in case 3 of proposition 1, it is optimal for the firm to set $\phi = 0$ and but in this case, consumers will will not choose to check.

Case 6: $\tilde{\alpha} \leq 1 - \frac{1}{\phi} \leq \alpha$ and $k \geq k^*(\alpha, \phi)$

The condition can be translated as follows: $\frac{1}{1-\tilde{\alpha}} \leq \phi \leq \frac{1-\sqrt{\alpha-2k}}{1-\alpha}$

$\Pi = \frac{5}{8} + \frac{1}{2}(1-(1-\alpha)\phi)\phi$

Based on Proposition 1, the local maximum point is at $\phi = \frac{1}{2(1-\alpha)}$. To make it the optimal penalty fee in this case, it also needs to satisfy the condition $\frac{1}{1-\tilde{\alpha}} \leq \phi \leq \frac{1-\sqrt{\alpha-2k}}{1-\alpha}$.

By comparison, it can be easily shown that

(i) $\frac{1}{2(1-\alpha)} > \frac{1}{1-\tilde{\alpha}}$ if $\alpha > \frac{1}{2}\tilde{\alpha} + \frac{1}{2}$

(ii) $\frac{1-\sqrt{\alpha-2k}}{1-\alpha} > \frac{1}{2(1-\alpha)}$ if $k > \frac{1}{2}\alpha - \frac{1}{8}$

(iii) $\frac{1-\sqrt{\alpha-2k}}{1-\alpha} > \frac{1}{1-\tilde{\alpha}}$ if $k > \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$

Moreover, $\frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} < \frac{1}{2}\alpha - \frac{1}{8}$ if $\alpha > \frac{1}{2}\tilde{\alpha} + \frac{1}{2}$

Therefore, we have the following conclusions:

(i) If $\frac{1}{1-\tilde{\alpha}} \leq \frac{1}{2(1-\alpha)} \leq \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$, the optimal penalty fee should be set at $\phi = \frac{1}{2(1-\alpha)}$ and the level of profit is $\Pi = \frac{5\alpha-6}{8(\alpha-1)}$. In this case, the condition can be written as $\alpha > \frac{1+\tilde{\alpha}}{2}$ and $k > \frac{1}{2}\alpha - \frac{1}{8}$

(ii) If $\frac{1}{2(1-\alpha)} < \frac{1}{1-\tilde{\alpha}} \leq \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$, the optimal penalty fee should be set at $\phi = \frac{1}{1-\tilde{\alpha}}$ and the level of profit is $\Pi = \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$. In this case, the condition can be written as $\alpha < \frac{1+\tilde{\alpha}}{2}$ and $k > \frac{1}{2}\alpha - \frac{1}{8}$

(iii) If $\frac{1}{1-\tilde{\alpha}} \leq \frac{1-\sqrt{-2k+\alpha}}{1-\alpha} < \frac{1}{2(1-\alpha)}$, the optimal penalty fee should be set at $\phi = \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$ and the level of profit is $\Pi = \frac{8k-9\alpha+5+4\sqrt{-2k+\alpha}}{8(1-\alpha)}$, In this case, the condition can be written as $\alpha > \frac{1+\tilde{\alpha}}{2}$ and $\frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} < k < \frac{1}{2}\alpha - \frac{1}{8}$

Case 7: $1 - \frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$and $\phi > 1$and $k \geq k^*(\alpha, \phi)$

First, the conditions can be reduced to $1 - \frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$and $\phi > 1$ and $1 < \phi <= \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$

$\Pi = 1 - \phi^2((1-\alpha)(1-\phi) + \frac{1}{8}(4\tilde{\alpha}^2 + (1-\tilde{\alpha}^2)(3+\tilde{\alpha}^2-4\alpha)\phi^2)))$ Based on Proposition 1, the optimal penalty fee should be charged at the extreme point $\phi = \frac{1}{1-\tilde{\alpha}}$. However, it also needs to satisfy the condition $1 - \frac{1}{\phi} < \tilde{\alpha} < \sqrt{\frac{\phi^2-2\phi+2}{\phi^2}}$ and $1 < \phi \leq \frac{\sqrt{2k-\alpha+1}}{1-\alpha}$

We also know that

(i) $\frac{1}{1-\tilde{\alpha}} > \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$ if $k < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$

(ii) $\frac{1-\sqrt{-2k+\alpha}}{1-\alpha} \geq 1$ if $k > \frac{1}{2}\alpha - \frac{1}{2}\alpha^2$

Therefore, we can easily get the following conclusions:

If $\frac{1}{1-\tilde{\alpha}} < \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$, it is optimal to set $\phi = \frac{1}{1-\tilde{\alpha}}$ and the level of profit is $\Pi = \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$. In this case, the condition can be written as $k > \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$.

If $\frac{1}{1-\tilde{\alpha}} > \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$, it is optimal to set $\phi = \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$ and the level of profit can be calculated. In this case, the condition can be written as $\frac{1}{2}\alpha - \frac{1}{2}\alpha^2 < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$.

Case 8: Otherwise

$\Pi = \frac{1}{2}$

Comparison of the cases above, the firm earns a highest level of profit from not tracking, then followed by consumers who expect not to track, but will use the tracking technology. Moreover, the firm earns a lowest level of profit if consumers plan to track their consumption.

With the optimal contract derived in each cases, We are able to compare different cases and find out the optimal contract that maximize firm's profit. In particular, We have

(i) $\frac{5\alpha-6}{8(\alpha-1)} > 1$ if $\alpha > \frac{2}{3}$

(ii) $\frac{5\alpha-6}{8(\alpha-1)} > \frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2}$ if $\alpha > \frac{1+\tilde{\alpha}}{2}$

(iii) $\frac{5\tilde{\alpha}^2-14\tilde{\alpha}+4\alpha+5}{8(\tilde{\alpha}-1)^2} > 1$ if $\alpha > \frac{3}{4}\tilde{\alpha}^2 - \frac{1}{2}\tilde{\alpha} + \frac{3}{4}$

(iv) $\frac{8k-9\alpha+4\sqrt{-2k+\alpha}+5}{8(\alpha-1)} > 1$ if $k > \frac{\alpha+1}{8} - \frac{\sqrt{3\alpha-2}}{4}$

(v) $\frac{8k-9\alpha+4\sqrt{-2k+\alpha}+5}{8(\alpha-1)} < \frac{5\alpha-6}{8(\alpha-1)}$always holds

(vi) $\Pi(\frac{1-\sqrt{\alpha-2k}}{1-\alpha}) > 1$ if $k > k_1$

Moreover, $\frac{\alpha+1}{8} - \frac{\sqrt{3\alpha-2}}{4} < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$ if $\alpha > \frac{1+\tilde{\alpha}}{2}$

Therefore, We can get the optimal penalty fee as follows,

If $k > \frac{1}{2}\alpha - \frac{1}{8}$ and $\alpha > \max\{\frac{1}{2}\tilde{\alpha} + \frac{1}{2}, \frac{2}{3}\}$, it is optimal to set $\phi = \frac{1}{2(1-\alpha)}$

If $\frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} < k < \frac{1}{2}\alpha - \frac{1}{8}$, it is optimal to set $\phi = \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$

If $k > \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$ and $\frac{3}{4}\tilde{\alpha}^2 - \frac{1}{2}\tilde{\alpha} + \frac{3}{4} < \alpha < \frac{1+\tilde{\alpha}}{2}$, it is optimal to set $\phi = \frac{1}{1-\tilde{\alpha}}$

If $\max\{k_1, \frac{1}{2}\alpha - \frac{1}{2}\alpha^2\} < k < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2}$, it is optimal to set $\phi = \frac{1-\sqrt{-2k+\alpha}}{1-\alpha}$

Otherwise, it is optimal to set $\phi = 0$

That is,

$$\phi^* = \begin{cases} \frac{1}{2(1-\alpha)} & \alpha > \max(\frac{2}{3}, \frac{1+\tilde{\alpha}}{2}) \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{8} \\ \frac{1}{1-\tilde{\alpha}} & -\frac{1}{2}\tilde{\alpha} + \frac{3}{4}\tilde{\alpha}^2 + \frac{3}{4} < \alpha < \frac{1+\tilde{\alpha}}{2} \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ \frac{1-\sqrt{-2k+\alpha}}{1-\alpha} & \frac{1}{2}\alpha - \frac{1}{8} > k \geq \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \text{ and } \alpha > \frac{1+\tilde{\alpha}}{2} \\ & \text{or} \max\{k_1, \frac{1}{2}\alpha - \frac{1}{2}\alpha^2\} < k < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ 0 & OW \end{cases}$$

where k_1 makes the firm indifferent from charging $\phi = 0$ and $\phi = \frac{1-\sqrt{\alpha-2k}}{1-\alpha}$,

$$p^* = \begin{cases} & \alpha > \max(\frac{2}{3}, \frac{1+\tilde{\alpha}}{2}) \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{8} \\ \frac{5}{8} & -\frac{1}{2}\tilde{\alpha} + \frac{3}{4}\tilde{\alpha}^2 + \frac{3}{4} < \alpha < \frac{1+\tilde{\alpha}}{2} \text{ and } k \geq \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ & \frac{1}{2}\alpha - \frac{1}{8} > k \geq \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \text{ and } \alpha > \frac{1+\tilde{\alpha}}{2} \\ p_c & \max\{k_1, \frac{1}{2}\alpha - \frac{1}{2}\alpha^2\} < \ k < \frac{1}{2}\alpha - \frac{1}{2}\frac{(\alpha-\tilde{\alpha})^2}{(\tilde{\alpha}-1)^2} \\ 1 & OW \end{cases}$$

$$p_c^* = 1 + \frac{1}{8(\alpha-1)^4}\Big(\begin{array}{c} 4(2\alpha^2 - 2\alpha^3 + \tilde{\alpha}^2 - \tilde{\alpha}^4 - \alpha - \alpha\tilde{\alpha}^2 + 2\alpha^2\tilde{\alpha}^2)\sqrt{\alpha - 2k} + 8k\alpha^2(\tilde{\alpha}^2 - 1) - 8k(\tilde{\alpha}^2 - \alpha)^2 \\ +4k\alpha(1-\tilde{\alpha}^4) + (4(\tilde{\alpha}^2\alpha)(1-\tilde{\alpha}^2)(\alpha-2k)^{\frac{3}{2}} + (2k-1)^2(\tilde{\alpha}^2-1)^2 \\ -4\alpha^3\beta^2 + 16\alpha^3 + \alpha^2\beta^4 - 10\alpha^2\beta^2 - 19\alpha^2 + 6\alpha\beta^4 + 14\alpha - 4 \end{array} \Big)$$

PROPOSITION 3: *If $\sigma > \frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)}$, the optimal penalty fee is $\phi_0^* = \frac{1}{2(1-(\sigma\alpha_L+(1-\sigma)\alpha_H))}$ without tracking option. With the advancement of consumption tracking technology and the cost associated with the tracking technology k is $k^*(\alpha_L, \phi_0^*) < k < k^*(\alpha_H, \phi_0^*)$,*

(i) the firm will increase the penalty fee to $\phi^* = \frac{1}{2(1-\alpha_H)}$ if $\sigma < \min(\sigma_2, 3\alpha_H - 2)$ and $k > \frac{1}{2}\alpha_H - \frac{1}{8}$. In this case, less forgetful consumers will use the tracking technology while more forgetful consumers will not;

(ii) if $\sigma < \min(\sigma_1, -\frac{(-8k+\alpha_H-4\sqrt{-2k+\alpha_H}+3)}{8k-4\alpha_H+4\sqrt{-2k+\alpha_H}})$ and $k < \frac{1}{2}\alpha_H - \frac{1}{8}$, the optimal penalty fee will be increased to $\frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$. In this case, less forgetful consumers will use the tracking technology while more forgetful consumers will not;

(iii) the firm will decrease the penalty fee to $\frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$ if $\frac{(-8k+2\alpha_L-4\alpha_H-4(1+\alpha_L-2\alpha_H)\sqrt{-2k+\alpha_L}+8k\alpha_H+3\alpha_L^2-4\alpha_L\alpha_H+3)}{4(\alpha_H-\alpha_L)(1-\sqrt{-2k+\alpha_L})^2} > \sigma > \max\{\sigma_2, \sigma_1\}$. In this case, neither group of consumers will use the tracking technology;

(iv) otherwise, the firm will set the penalty fee to be 0. In this case, neither group of consumers will use the tracking technology

Proof:

The proportion of less forgetful consumers α_L is σ, and the rest of consumers are more forgetful consumers α_H. Moreover, both groups of consumers are assumed to have $\alpha_{L,H} > \max\{\frac{2}{3}, \frac{1+\tilde{\alpha}}{2}\}$, implying that consumers are sufficiently forgetful or have a relative low sophistication. As explained before, consumers share the same perception, suggesting that they have the same willingness to pay. Therefore, the firm has to provide one contract to serve both groups, instead of conducting price discrimination. It is notable that the discussion is based on the assumption that $k > \frac{1}{2}\tilde{\alpha}$, which makes sure that consumers will make the purchase. If the tracking cost is lower than $\frac{1}{2}\tilde{\alpha}$, the firm should just provide a contract with zero penalty fee and high sign-up price.

Without tracking option

According to the analysis in Proposition 1, if $\tilde{\alpha} < 1 - \frac{1}{\phi} < \alpha_L$, both groups of consumers will expect zero probability of incurring a penalty fee and have the same willingness to pay ($p = \frac{5}{8}$). However, the probability of incurring a penalty fee, which depends on their levels of forgetfulness, would be different. Therefor, the firm's profit can be written as follows:

$$\pi = \sigma(\frac{5}{8} + \frac{1}{2}(1-(1-\alpha_L)\phi)\phi) + (1-\sigma)(\frac{5}{8} + \frac{1}{2}(1-(1-\alpha_H)\phi)\phi)$$

By maximizing the firm's profit, the local maximum is at $\phi = \frac{1}{2(1-(1-\sigma)\alpha_H-\sigma\alpha_L)}$. However, it has to fall

into the region defined earlier $\frac{1}{1-\tilde{\alpha}} < \phi < \frac{1}{1-\alpha_L}$. Therefore, if $\sigma > \frac{\alpha_1 - 2\alpha_H + 1}{2\alpha_L - 2\alpha_H} = \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H - \alpha_L)}$, it is optimal for the firm to set the penalty fee as $\phi = \frac{1}{2(1-(1-\sigma)\alpha_H - \sigma\alpha_L)}$ and the corresponding level of profit is $\Pi = \frac{5}{8} + \frac{1}{8(1-(1-\sigma)\alpha_H - \sigma\alpha_L)}$. The assumption about $\alpha_{H,L}$ ensures that it is the optimal penalty fee that maximizes firm's profit. It is also higher than the case where the firm only focusing on more forgetful consumers.

However, If $\sigma < \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H - \alpha_L)}$, it implies that $\phi = \frac{1}{2(1-(1-\sigma)\alpha_H - \sigma\alpha_L)} > \frac{1}{(1-\alpha_L)}$. It also suggests that $\frac{1}{2(1-\alpha_H)}$ is also higher than $\frac{1}{1-\alpha_L}$. Instead of earning profit from both groups, the firm would rather only focusing on more forgetful consumers, and sets $\phi = \frac{1}{2(1-\alpha_H)}$. In this case, less forgetful consumers will never incur a penalty fee. The rationale of this contract is that if the proportion of less forgetful consumers is too small, the penalty fee becomes too high for less forgetful consumers such that they will never choose to consume in period two, making the firm loses the penalty fee revenue from them, but it gets compensated by the higher penalty fee revenue from more forgetful consumers. In this case, less forgetful consumers get cross subsidy from more forgetful consumers and becomes better off.

In the paper, we focus on the case where there are enough less forgetful consumers in the market ($\sigma > \frac{1}{2} - \frac{1-\alpha_H}{2(\alpha_H - \alpha_L)}$). Then the optimal contract can be written as follows: $p = \frac{5}{8}$, and $\phi = \frac{1}{2(1-(1-\sigma)\alpha_H - \sigma\alpha_L)}$.

With consumption tracking technology

Let's first check the threshold for using the tracking technology.

According to Lemma 2, facing the penalty fee described, the threshold for using the tracking technology is $k^*(\alpha, \phi) = \frac{1}{2}\alpha - \frac{1}{2}(1-(1-\alpha)\phi)^2$. Therefore, for consumers with different levels of forgetfulness, their thresholds can be expressed as follows:

In the heterogeneous market, $k^*(\alpha_L, \phi_0^*) = \frac{1}{2}\alpha_L - \frac{1}{2}(\frac{\alpha_L - 1}{2\alpha_H + 2\sigma\alpha_L - 2\sigma\alpha_H - 2} - 1)^2$ and

$k^*(\alpha_H, \phi_0^*) = \frac{1}{2}\alpha_H - \frac{1}{2}(\frac{\alpha_H - 1}{2\alpha_H + 2\sigma\alpha_L - 2\sigma\alpha_H - 2} - 1)^2$

If $\sigma > \frac{1 - 2\alpha_H + \sqrt{\alpha_L + \alpha_H - 1}}{2(\alpha_L - \alpha_H)}$, we will have $k^*(\alpha_H, \phi_0^*) < k^*(\alpha_L, \phi_0^*)$, implying that facing a penalty fee ϕ_0^*, less forgetful consumers are more likely to track their consumption. There are three cases depending on the level of tracking cost.

1) $k > k^*(\alpha_L, \phi_0^*)$

Neither group of consumers will use the tracking technology. As a result, the firm does not need to change its contract despite the advancement of consumption tracking technology.

2) if $k < k^*(\alpha_H, \phi_0^*)$,

Without price adjustment, both groups will track their assumption and be able to avoid paying penalty fee and the firm only gets the sign-up price. Hence, both group of consumers will be better off and the level of profit earned by the firm is $\Pi = \frac{5}{8}$

In this case, we have

$\frac{1}{2(1-\alpha_H)} > \phi_0^* > -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H - 1} > -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L - 1}$

There are 3 cases when the firm considers to change its contract,

First, if the firm decides to serve both group of consumers, optimal to set $\phi = -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L - 1}$ and the level of profit is $\Pi = \frac{5}{8} + \frac{4(\alpha_H - \alpha_L)(1-\sigma) - 4(1-\sigma\alpha_L - (1-\sigma)\alpha_H)(\alpha_L - 2k) + 4((1-2\sigma)(\alpha_L - \alpha_H) + 1 - \alpha_H)\sqrt{\alpha_L - 2k}}{8(1-\alpha_L)^2}$

Second, if the firm decides to only focus on the more forgetful consumers. It will set $\phi = -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H - 1}$. In this case, less forgetful consumers will track their consumption while the more forgetful consumers will not. Therefore, the firm only earns penalty fee revenue from more forgetful consumers. And the level of corresponding profit is $\Pi = \frac{5}{8} - \frac{4(1-\sigma)(\sqrt{\alpha_H - 2k} - \alpha_H + 2k)}{8(\alpha_H - 1)}$

Third, the firm can always choose to provide a contract with zero penalty fee and high sign-up price while making sure both group of consumers will choose to make a purchase.

Through comparing the three cases, We obtain the following results

$$\phi^* = \begin{cases} -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1} & \sigma < \min(\sigma_1, -\frac{(-8k+\alpha_2-4\sqrt{-2k+\alpha_H}+3)}{8k-4\alpha_H+4\sqrt{-2k+\alpha_H}}) \\ -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L-1} & -\frac{(-8k+2\alpha_L-4\alpha_H-4\sqrt{-2k+\alpha_L}+8k\alpha_H+3\alpha_L^2-4\alpha_L\alpha_H-4\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}+3)}{4\alpha_L-4\alpha_H-8k\alpha_L+8k\alpha_H+4\alpha_L^2-4\alpha_L\alpha_H-8\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}} > \sigma > \sigma_1 \\ 0 & OW \end{cases}$$

Where $\sigma_1 = \frac{(\alpha_H-1)(8k-8\alpha_L+4\alpha_H+4(1+\alpha_L+2\alpha_H)\sqrt{-2k+\alpha_L}+5(\alpha_L-1)^2+4\alpha_H(-2k+\alpha_L)+5)+(\alpha_L-1)^2(8k-9\alpha_H+4\sqrt{-2k+\alpha_H})}{-4(\alpha_H-1)(\alpha_L-\alpha_H)(1-2k+\alpha_L-2\sqrt{-2k+\alpha_L})+(\alpha_L-1)^2(8k-4\alpha_H+4\sqrt{-2k+\alpha_H})}$

The intuition behind this contract is as follows: if the proportion of less forgetful consumers is small, the firm will choose to focus on more forgetful consumers since those consumers gives the firm higher profit. In equilibrium, less forgetful consumers will use the tracking technology and avoid paying penalty fee, while more forgetful consumers will not use the tracking technology. However, if the proportion of less forgetful consumers is higher, the firm will lower the penalty fee to prevent both group of consumers from using the tracking technology. In equilibrium, neither group of consumers will use the tracking technology.

3) $k^*(\alpha_H, \phi_0^*) < k < k^*(\alpha_L, \phi_0^*)$

In this case, less forgetful consumers will choose to track their consumption while the more forgetful consumers will not do so. Without price adjustment, less forgetful consumers will be better off since they could avoid paying high penalty fee.

In this case, we have the following relationship:

$-\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L-1} < \phi_0^* < -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1}$

There are 3 possible cases when the firm decides its penalty fee.

First, if the firm only focuses on more forgetful consumers, it will set the penalty fee such that more forgetful consumers will not use the tracking technology. Therefore, it is optimal for the firm to set $\phi = \frac{1}{2(1-\alpha_H)}$ if $k^*(\alpha_L, \phi_0^*) > k > \frac{1}{2}\alpha_H - \frac{1}{8}$. The corresponding level of profit is $\Pi = \frac{\sigma+5\alpha_H-6}{8(\alpha_H-1)}$

If $k^*(\alpha_H, \phi_0^*) < k < \frac{1}{2}\alpha_H - \frac{1}{8}$, it is optimal to set $\phi = -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1}$ and the level of profit is $\Pi = \frac{5}{8} - \frac{4(1-\sigma)(\sqrt{\alpha_H-2k}-\alpha_H+2k)}{8(\alpha_H-1)}$

Second, the firm reduce its penalty fee such that neither group of consumers will use the tracking technology. In this case, it is optimal to set $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_1}$, and the corresponding level of profit is $\Pi = \frac{5}{8} + \frac{4(\alpha_H-\alpha_L)(1-\sigma)-4(1-\sigma\alpha_L-(1-\sigma)\alpha_H)(\alpha_L-2k)+4((1-2\sigma)(\alpha_L-\alpha_H)+1-\alpha_H)\sqrt{\alpha_L-2k}}{8(1-\alpha_L)^2}$ Third, the firm can always choose to provide a contract with zero penalty fee and gets a level of profit at 1.

Comparing the three cases, We get the following results:

$$\phi^* = \begin{cases} \frac{1}{2(1-\alpha_H)} & k > \frac{1}{2}\alpha_H - \frac{1}{8}, \\ & \sigma < \min(\sigma_2, 3\alpha_H - 2) \\ -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1} & k < \frac{1}{2}\alpha_H - \frac{1}{8} \\ & \sigma < \min(\sigma_1, -\frac{(-8k+\alpha_H-4\sqrt{-2k+\alpha_H}+3)}{8k-4\alpha_H+4\sqrt{-2k+\alpha_H}}) \\ -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L-1} & -\frac{(-8k+2\alpha_L-4\alpha_H-4\sqrt{-2k+\alpha_L}+8k\alpha_H+3\alpha_L^2-4\alpha_L\alpha_H-4\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}+3)}{4\alpha_L-4\alpha_H-8k\alpha_L+8k\alpha_H+4\alpha_L^2-4\alpha_L\alpha_H-8\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}} > \sigma > \max\{\sigma_2, \sigma_1\} \\ 0 & OW \end{cases}$$

Where $\sigma_1 = \frac{(\alpha_H-1)(8k-8\alpha_L+4\alpha_H+4(1+\alpha_L+2\alpha_H)\sqrt{-2k+\alpha_L}+5(\alpha_L-1)^2+4\alpha_H(-2k+\alpha_L)+5)+(\alpha_L-1)^2(8k-9\alpha_H+4\sqrt{-2k+\alpha_H})}{-4(\alpha_H-1)(\alpha_L-\alpha_H)(1-2k+\alpha_L-2\sqrt{-2k+\alpha_L})+(\alpha_L-1)^2(8k-4\alpha_H+4\sqrt{-2k+\alpha_H})}$

and

$\sigma_2 = -\frac{(1-\alpha_H)(8k-18\alpha_L+4\alpha_H+4\sqrt{-2k+\alpha_L}+5\alpha_L^2+4\alpha_H(-2k+\alpha_L)+4\alpha_L\sqrt{-2k+\alpha_L}-8\alpha_H\sqrt{-2k+\alpha_L}+5)+(5\alpha_H-6)(\alpha_L-1)^2}{(1-\alpha_L)^2+(1-\alpha_H)(4\alpha_L-4\alpha_H+4\alpha_L(-2k+\alpha_L)-4\alpha_H(-2k+\alpha_L)-8\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L})}$

The intuition behind this contract is straightforward: (1) when the proportion of less forgetful consumers will small, it is optimal for the firm to *increase* the penalty fee and concentrates on more forgetful consumers. As for the optimal penalty fee, it depends on how big the cost of using the consumption tracking technology. In equilibrium, less forgetful consumers will track their consumption while the more forgetful consumers will not. (2) when the proportion of consumers are high, it is optimal for the firm to *decrease* its penalty fee such that neither group of consumers will choose to track their consumption.

With the condition that $\frac{1-2\alpha_H+\sqrt{\alpha_L+\alpha_H-1}}{2(\alpha_L-\alpha_H)} > \sigma > \frac{\alpha_L-2\alpha_H+1}{2\alpha_L-2\alpha_H}$, less forgetful consumers are less likely to use the tracking technology. Therefore, how the firm designs its contract depends on the level of tracking cost associated with the consumption tracking technology.

Case 1: $k > k^*(\alpha_H, \phi_0^*)$

Without price adjustment, neither group will track their consumption.

With price adjustment, the optimal penalty fee will not change since consumers will not use the tracking technology. The advancement of tracking technology does not affect consumer's behavior and firm's pricing, as well as market equilibrium.

Case 2: $k < k^*(\alpha_L, \phi_0^*)$

Without price adjustment, both groups will track their consumption. In that case, the firm loses its revenue from penalty fees and only earns the sign-up price from consumers. And the firm's profit becomes $\Pi = \frac{5}{8}$.

In this case, $\frac{1}{2(1-\alpha_H)} > \phi_0^* > -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1} > -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L-1}$

If the firm is allowed to change its contract, the firm can set the penalty fee such that the less forgetful consumers will not choose to use the technology, that is $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$. In this case, neither groups of forgetful consumers will not track their consumption. The corresponding level of profit is

$\Pi = \frac{5}{8} + \frac{4(\alpha_H-\alpha_L)(1-\sigma)-4(1-\sigma\alpha_L-(1-\sigma)\alpha_H)(\alpha_L-2k)+4((1-2\sigma)(\alpha_L-\alpha_H)+1-\alpha_H)\sqrt{\alpha_L-2k}}{8(1-\alpha_L)^2}$, which will be lower as the proportion of less forgetful consumers increases.

The firm could also choose only to focus on the more forgetful consumers consumers. However, in this case, the less forgetful consumers will choose to use the tracking technology.

Therefore, it could be optimal for the firm to set $\phi = \frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$ and the corresponding level of profit is $\Pi = \frac{5}{8} - \frac{4(1-\sigma)(\sqrt{\alpha_H-2k}-\alpha_H+2k)}{8(\alpha_H-1)}$

Alternatively, the firm could always choose to provide a contract with zero penalty fee and a high sign-up price, thus earns a level of profit at 1.

By comparing different case, We obtain the following result:

$$\phi^* = \begin{cases} -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1} & \sigma < \min(\sigma_1, -\frac{(-8k+\alpha_H-4\sqrt{-2k+\alpha_H}+3)}{8k-4\alpha_H+4\sqrt{-2k+\alpha_H}}) \\ -\frac{1-\sqrt{-2k+\alpha_L}}{\alpha_L-1} & -\frac{(-8k+2\alpha_L-4\alpha_H-4\sqrt{-2k+\alpha_L}+8k\alpha_H+3\alpha_L^2-4\alpha_L\alpha_H-4\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}+3)}{4\alpha_L-4\alpha_H-8k\alpha_L+8k\alpha_H+4\alpha_L^2-4\alpha_L\alpha_H-8\alpha_L\sqrt{-2k+\alpha_L}+8\alpha_H\sqrt{-2k+\alpha_L}} > \sigma > \sigma_1 \\ 0 & OW \end{cases}$$

Where $\sigma_1 = \frac{(\alpha_H-1)(8k-8\alpha_L+4\alpha_H+4(1+\alpha_L+2\alpha_H)\sqrt{-2k+\alpha_L}+5(\alpha_L-1)^2+4\alpha_H(-2k+\alpha_L)+5)+(\alpha_L-1)^2(8k-9\alpha_H+4\sqrt{-2k+\alpha_H})}{-4(\alpha_H-1)(\alpha_L-\alpha_H)(1-2k+\alpha_L-2\sqrt{-2k+\alpha_L})+(\alpha_L-1)^2(8k-4\alpha_H+4\sqrt{-2k+\alpha_H})}$

The intuition is the same as before.

Case 3: $k^*(\alpha_L,\phi_0^*) < k < k^*(\alpha_H,\phi_0^*)$

Without price adjustment, more forgetful consumers consumers will track while less forgetful consumers will not do so. Therefore, the firm will lose the revenue from more forgetful consumers.

If the firm is allowed to change its contract, it should simply reduce the penalty fee such that more forgetful consumers will not choose to use the tracking technology. That is, $\phi = \frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$. Moreover, $k^*(\alpha_L,\phi_0^*) < k < k^*(\alpha_H,\phi_0^*)$ makes sure that the new penalty fee to be smaller than $\frac{1}{2(1-(1-\sigma)\alpha_H-\sigma\alpha_L)}$. In summary, the optimal penalty fee should be set at $\phi = \frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$, and the level of profit is

$\Pi = \frac{5}{8} + \frac{4(\alpha_H-1)(\alpha_H-2k)+4\sigma(1+\alpha_H-2k)(\alpha_L-\alpha_H)+4(1-\alpha_H+2\sigma\alpha_H-2\sigma\alpha_L)\sqrt{\alpha_H-2k}}{8(\alpha_H-1)^2}$, which decreases as σ increases. In this case, neither group of consumers will choose to track their consumption.

The firm could also choose to provide a contract with zero penalty fee and high sign-up price.

By comparing different cases, We obtain the following results:

$$\phi^* = \begin{cases} -\frac{1-\sqrt{-2k+\alpha_H}}{\alpha_H-1} & \sigma < -\frac{(1-\alpha_H)(8k+4\sqrt{\alpha_H-2k}-\alpha_H-3)}{4(\alpha_L-\alpha_H)(\sqrt{\alpha_H-2k}-1)^2} \\ 0 & OW \end{cases}$$

PROPOSITION 4: *If the firm decreases its penalty fee, both groups of consumers are more likely to incur a penalty fee. Moreover, despite neither groups of consumers use the consumption tracking technology, less forgetful consumers are worse off by paying more penalty, while the more forgetful consumers are better off when* $-\frac{\alpha_L(\alpha_L^2-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1)}{(\alpha_L+1)^2(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)} + \frac{1}{2}\alpha_L - \frac{1}{8}(\frac{\alpha_L-2\sigma\alpha_L-2\alpha_H+2\sigma\alpha_H+1}{\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1})^2 < k < k^*(\alpha_L,\phi_0^*)$. *However, if the firm increases its penalty fee and only focuses on more forgetful consumers, less forgetful consumers will track their consumption and become better off while more forgetful consumers will not track and become worse off.*

Proof:

If $k(\alpha_L,\phi_0^*) > k(\alpha_H,\phi_0^*)$, more forgetful consumers are less likely to use the tracking technology. There are three cases depending on the level of tracking cost.

Case 1: $k > k^*(\alpha_L,\phi_0^*)$

If the tracking cost k associated with the tracking technology is greater than $k^*(\alpha_L,\phi_0^*)$, the optimal penalty fee is the same as the case of no tracking technology. In that case, the advancement of consumption tracking technology does not consumer behavior and firm profit.

Case 2: If $k < k^*(\alpha_H,\phi_0^*)$

If $k < k^*(\alpha_H,\phi_0^*)$, the advancement of consumption tracking technology makes both forgetful consumers to use it. Based on the discussion above, if the proportion of less forgetful consumers is big such that the firm decides to reduce the penalty fee to prevent both group of consumers from using the tracking technology.

If $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$

$CS(\alpha_L) = \frac{2k-\alpha_1+2k\alpha_L-\alpha_L^2+2\alpha_L\sqrt{\alpha_L-2k}}{4\alpha_L-4}$

$CS(\alpha_H) = \frac{(\alpha_1+\alpha_H-\sqrt{\alpha_L-2k}-\alpha_H\sqrt{\alpha_L-2k})(\alpha_L-\alpha_H-\sqrt{\alpha_L-2k}+\alpha_H\sqrt{\alpha_L-2k})}{4(\alpha_1-1)^2}$

$\Pi = \frac{5}{8} + \frac{4(\alpha_H-\alpha_L)(1-\sigma)-4(1-\sigma\alpha_L-(1-\sigma)\alpha_H)(\alpha_L-2k)+4((1-2\sigma)(\alpha_L-\alpha_H)+1-\alpha_H)\sqrt{\alpha_L-2k}}{8(1-\alpha_L)^2}$

If $\phi = \frac{1}{1-2(1-\sigma)\alpha_H-\sigma\alpha_L)}$

$CS(\alpha_L) = -\frac{(\alpha_L-2\alpha_2-2\sigma\alpha_L+2\sigma\alpha_H+1)(\alpha_L+2\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{16(\alpha_H+\sigma\alpha_1-\sigma\alpha_H-1)^2}$
$CS(\alpha_H) = \frac{1}{16}\frac{3\alpha_H+2\sigma\alpha_1-2\sigma\alpha_H-1}{(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)^2}(\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)$
$\Pi = \frac{5}{8} + \frac{1}{8(1-(1-\sigma)\alpha_H-\sigma\alpha_L)}$

It is easy to show that firm's profit decreases in this case. Moreover, both groups of consumers become better off.

Case 3: If $k^*(\alpha_H,\phi_0^*) < k < k^*(\alpha_L,\phi_0^*)$

Without price adjustment, less forgetful consumers will use the tracking technology and become better off due to the advancement. While the more forgetful consumers will not use the tracking technology. Hence, less forgetful consumers will be better off and the firm lose the penalty fee revenue from those consumers.

When the firm is allowed to use the tracking technology, based on the discussion above, due to the advancement of the consumption tracking technology, the firm will either increase or decrease its penalty fee, depending on the relative size of less forgetful consumers. In particular, if the proportion of less forgetful consumers is small, facing the advancement of consumption tracking technology, the firm will increase its penalty fee and only make sure that more forgetful consumers will not use the tracking technology. In this case, less forgetful consumers will be better off since they will use the tracking technology and avoid paying penalty fees. But more forgetful consumers will be worse off since the new penalty fee faced is higher and closer to ϕ_H^*.

If there are enough less forgetful consumers, the firm will decrease the penalty fee and prevent both groups of consumers from using it. In the following analysis, the We will focus on the case where the firm decreases the penalty fee. Compare to the no tracking option, the profit with tracking technology is lower. However, the difference (loss) in profit will be lower as the tracking cost decreases.

It is easy to sow that both groups are more likely to incur a penalty fee, but less forgetful consumers are expected to pay more on penalty fees if $\frac{1}{2}\alpha_L - \frac{1}{2}(\frac{\alpha_1-1}{2\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-2}-1)^2 + \frac{1}{2}(\sigma-1)\frac{\alpha_L-\alpha_H}{\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1} < k < \frac{1}{2}\alpha_1 - \frac{1}{2}(\frac{\alpha_L-1}{2\alpha_H+2\sigma\alpha_1-2\sigma\alpha_H-2}-1)^2$

As for realized consumer surplus, it can be computed through the formula below:

$CS = \frac{1}{2}(\frac{1+\frac{1}{2}}{2} + (1-(1-\alpha_L)\phi)(\frac{1+(1-\alpha_L)\phi}{2}-\phi)) + \frac{1}{2}(\frac{1}{2}) - \frac{5}{8}$
If $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$
$CS(\alpha_L) = \frac{1}{4\alpha_L-4}(2k-\alpha_1+2k\alpha_L-\alpha_L^2+2\alpha_L\sqrt{\alpha_L-2k})$
$CS(\alpha_H) = \frac{1}{4(\alpha_L-1)^2}(\alpha_L-\alpha_H-\sqrt{\alpha_L-2k}+\alpha_H\sqrt{\alpha_L-2k})(\alpha_L+\alpha_H-\sqrt{\alpha_L-2k}-\alpha_H\sqrt{\alpha_L-2k})$
If $\phi = \frac{1}{1-2(1-\sigma)\alpha_H-\sigma\alpha_L)}$
$CS(\alpha_L) = -\frac{(\alpha_L-2\alpha_H-2\sigma\alpha_L+2\sigma\alpha_H+1)(\alpha_L+2\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{16(\alpha_H+\sigma\alpha_1-\sigma\alpha_H-1)^2}$
$CS(\alpha_H) = \frac{1}{16}\frac{(3\alpha_H+2\sigma\alpha_1-2\sigma\alpha_H-1)(\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)^2}$

The difference in consumer surplus increase with the proportion of less forgetful consumers. When we compare the differences in consumer welfare, it is easy to see that the difference increases as the proportion of less forgetful consumers increases. Moreover, consumer welfare decreases as if $-\frac{\alpha_L(\alpha_L^2-2\sigma\alpha_L-2\alpha_2+2\sigma\alpha_H+1)}{(\alpha_L+1)^2(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)} + \frac{1}{2}\alpha_L - \frac{1}{8}(\frac{\alpha_L-2\sigma\alpha_1-2\alpha_H+2\sigma\alpha_H+1}{\alpha_H+\sigma\alpha_1-\sigma\alpha_H-1})^2 < k < k^*(\alpha_L,\phi_0^*)$. As for more forgetful consumers, they will always be better off.

If $k(\alpha_L,\phi_0^*) < k(\alpha_H,\phi_0^*)$**,** more forgetful consumers are more likely to use the tracking technology. There are three cases depending on the level of tracking cost.

Case 1: $k > k^*(\alpha_H, \phi_0^*)$

If the tracking cost k associated with the tracking technology is greater than $k^*(\alpha_H, \phi_0^*)$, the optimal penalty fee is the same as the case of no tracking technology. In that case, the advancement of consumption tracking technology does not consumer behavior and firm profit.

Case 2: If $k < k^*(\alpha_L, \phi_0^*)$

If $k < k^*(\alpha_L, \phi_0^*)$, the advancement of consumption tracking technology makes more forgetful consumers to use it and the firm will respond by reducing the penalty fee to prevent them from doing it. Therefore, in the new equilibrium, $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$. Comparing with no tracking option case, both groups of consumers pay less on penalty fee and consumer welfare improves. In particular,

If $\phi = \frac{1-\sqrt{-2k+\alpha_L}}{1-\alpha_L}$

$CS(\alpha_L) = \frac{2k-\alpha_1+2k\alpha_L-\alpha_L^2+2\alpha_L\sqrt{\alpha_L-2k}}{4\alpha_L-4}$

$CS(\alpha_H) = \frac{(\alpha_1+\alpha_H-\sqrt{\alpha_L-2k}-\alpha_H\sqrt{\alpha_L-2k})(\alpha_L-\alpha_H-\sqrt{\alpha_L-2k}+\alpha_H\sqrt{\alpha_L-2k})}{4(\alpha_1-1)^2}$

$\Pi = \frac{5}{8} + \frac{4(\alpha_H-\alpha_L)(1-\sigma)-4(1-\sigma\alpha_L-(1-\sigma)\alpha_H)(\alpha_L-2k)+4((1-2\sigma)(\alpha_L-\alpha_H)+1-\alpha_H)\sqrt{\alpha_L-2k}}{8(1-\alpha_L)^2}$

If $\phi = \frac{1}{1-2(1-\sigma)\alpha_H-\sigma\alpha_L)}$

$CS(\alpha_L) = -\frac{(\alpha_L-2\alpha_2-2\sigma\alpha_L+2\sigma\alpha_H+1)(\alpha_L+2\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{16(\alpha_H+\sigma\alpha_1-\sigma\alpha_H-1)^2}$

$CS(\alpha_H) = \frac{1}{16}\frac{3\alpha_H+2\sigma\alpha_1-2\sigma\alpha_H-1}{(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)^2}(\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)$

$\Pi = \frac{5}{8} + \frac{1}{8(1-(1-\sigma)\alpha_H-\sigma\alpha_L)}$

By comparing the consumer surplus, it is easy to find that firm's profit decreases while consumer's welfare improves in this case. Moreover, the improvement in consumer welfare will be higher as tracking cost decreases. However, the improvement in consumer welfare will be mitigated (diluted) as the proportion of less forgetful consumers increases.

Case 3: If $k^*(\alpha_L, \phi_0^*) < k < k^*(\alpha_H, \phi_0^*)$

In terms of firm's profit,

if $\phi = \frac{1}{1-2(1-\sigma)\alpha_H-\sigma\alpha_L)}$

$CS(\alpha_L) = -\frac{(\alpha_L-2\alpha_H-2\sigma\alpha_L+2\sigma\alpha_H+1)(\alpha_L+2\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{16(\alpha_H+\sigma\alpha_1-\sigma\alpha_H-1)^2}$

$CS(\alpha_H) = \frac{1}{16}\frac{(3\alpha_H+2\sigma\alpha_1-2\sigma\alpha_H-1)(\alpha_H+2\sigma\alpha_L-2\sigma\alpha_H-1)}{(\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1)^2}$

$\Pi = \frac{1}{8}\frac{5\alpha_H+5\sigma\alpha_L-5\sigma\alpha_H-6}{\alpha_H+\sigma\alpha_L-\sigma\alpha_H-1}$

if $\phi = \frac{1-\sqrt{-2k+\alpha_H}}{1-\alpha_H}$

$CS(\alpha_L) = -\frac{(\alpha_L-\alpha_L\sqrt{\alpha_H-2k}+\alpha_H-\sqrt{\alpha_H-2k})(\alpha_L-\alpha_L\sqrt{\alpha_H-2k}-\alpha_H+\sqrt{\alpha_H-2k})}{4(\alpha_H-1)^2}$

$CS(\alpha_H) = \frac{2k-\alpha_H+2k\alpha_H-\alpha_H^2+2\alpha_H\sqrt{\alpha_H-2k}}{4\alpha_H-4}$

$\Pi = \frac{5}{8} + \frac{4(\alpha_H-1)(\alpha_H-2k)+4\sigma(1+\alpha_H-2k)(\alpha_L-\alpha_H)+4(1-\alpha_H+2\sigma\alpha_2-2\sigma\alpha_L)\sqrt{\alpha_H-2k}}{8(\alpha_H-1)^2}$, The firm's profit increases with k, implying that a higher tracking cost will increase the firm's profit.

Compare to the no tracking option, the profit with tracking technology is lower. However, the difference (loss) in profit will be lower as the tracking cost decreases. Facing a lower price, both groups of consumers are more likely to incur a penalty fee.

When we compare the differences in consumer welfare, it is easy to see that the difference increases as the proportion of less forgetful consumers increases. Moreover, consumer welfare decreases as if $\frac{\alpha_H}{2} - \frac{(\alpha_H-\alpha_L^2)^2}{2(\alpha_L^2-1)^2} < k < k^*(\alpha_H, \phi_0^*)$. As for more forgetful consumers, they will always be better off.