

MARKET DESIGN FOR GAMING PLATFORMS: USING STOPPING BEHAVIOR IN MATCHING ALGORITHMS

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Abstract

We investigate market design for online gaming platforms. Since a large part of such platforms' income is generated by advertisement, it is essential to know what influences how long users stay on the platform. We provide evidence of history-dependent stopping behavior in non-monetary environments. We find that there are two types of people: those who are more likely to stop playing after a loss; and others, who are more likely to keep playing until a win. We find that an individual's type is time-invariant over the years. We propose a behavioral dynamic choice model where utility from playing another game is directly affected by the outcome of the previous game. We structurally estimate this time non-separable preference model and conduct counterfactual analyses to evaluate alternative market designs. We find that in the context of online chess games a matching algorithm that incorporates stopping behavior can increase the length of play by 5.44%.

JEL Classification: D9, C5, C13, D4;

Keywords: time non-separable preferences, history dependent types, chess.com.

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1 Introduction

The online gaming industry grossed \$137.9 billion in 2018 and is predicted to reach \$180 billion by 2021.¹ In this industry, a significant part of revenue is related to the time that users spend on the platform through advertising. To maximize profit, a key question these marketplaces face is: how can one extend the time users stay on the platform? In this paper, we show that this goal can be achieved by a market design that leverages information about users' stopping behavior. We provide evidence that different users react differently to wins and losses when they decide whether to play another game or not, and show that this information can be used to design a matching algorithm to extend the duration of play.

We collect the data from the most prominent online chess platform, chess.com, which has over 30 million users and hosts, on average, 3 million chess games every day. We scraped the entire history of play for a random sample of about 20,000 users in the years of 2017 and 2018. Importantly, the data include the time and duration of each game. To keep the games homogeneous, we focus on “blitz”, the most popular type of chess game.² Based on the data for 2017, we find that 45% of players are *loss-stoppers* (players who are substantially more likely to stop playing after a loss) and 25% are *win-stoppers* (players who are substantially more likely to stop playing after a win). The remaining 30% are *neutral* types. We use the 2018 data to reclassify the same players and find that only 2% of loss-stoppers became win-stoppers and 7% of win-stoppers became loss-stoppers. That is, individuals are, in their vast majority, stable over time in terms of how they react to losses and wins.

We develop a structural model that allows for time non-separable preferences, where *future* game utility can depend on the history of play. Our estimates from the structural model are consistent with the above mentioned reduced form evidence. For some people, loss in a given game decreases the utility from playing another game. For others, it increases utility from playing another game.³

We then use our findings to propose a matching algorithm that increases the number of games played. Currently, the platform only uses players' ratings to match similar rated

¹ See: <https://www.broadbandsearch.net/blog/online-gaming-statistics>

² In blitz chess, each player has 3 to 15 minutes for the entire game.

³ Our classification may best be understood not as a reference to a fundamental, underlying bias, but as a reference to a reduced-form phenomenon. It is worth noting that several different underlying psychological forces could generate this behavior. See the discussion at the end of Section 3.1.

players with higher probability, ignoring additional information such as a player’s “type.”⁴ The matching mechanism we put forward leverages the following simple observation: loss-stoppers play more when they win, while win-stoppers play more when they lose. Thus, loss-stoppers can be matched with relatively lower rated individuals, so that they have a higher likelihood of winning and a higher likelihood of continuing to play. On the other hand, win-stoppers can be matched with players with higher ratings, so that they are more likely to lose and start one more game. We use our structural model to conduct counterfactual analyses. We show that incorporating user’s type can lead to a 5.44% increase in the number of games played with only minimally changing the user experience.⁵

Fundamentally, this paper presents and estimates a dynamic discrete choice model in which the agent may have non-separable preferences over the stochastic outcomes of their actions. In that sense, the application is analogous to the optimal stopping problems faced, for example, by taxi drivers, whose decisions to end their shifts may be influenced by their recent fares. Recent empirical research on this topic is complicated by spatial search frictions and limited by the imperfect observability of the decision makers’ identities and histories of outcomes.⁶ In contrast, we perfectly observe actions, payoff relevant outcomes, and independent realizations of each agent’s decision problem. We take advantage of this rich data to demonstrate that the players’ decisions cannot be reconciled in a model without non-separable preferences and that there is substantial heterogeneity in preferences across players. Though our quantitative findings clearly do not speak to the design of taxi markets, our counterfactual analysis provides an illustrative example of a policy that leverages such preference heterogeneity to further the market designer’s goals.⁷

In the context of chess games, [Anderson and Green \(2018\)](#) show that players are more likely to stop playing when they set a new personal best rating. For the purpose of revenue maximization, these patterns are not useful to design a better market as such events are very rare.⁸

The rest of the paper is organized as follows. Section 2 provides details on the data

⁴ The rating system reflects how well a person plays chess.

⁵ User behavior may change if we make changes to the matching algorithm. In this paper, we abstract from this concern. With the platform’s cooperation, an RCT can address these questions.

⁶ See [Camerer et al. \(1997\)](#); [Farber \(2005, 2008, 2015\)](#); [Crawford and Meng \(2011\)](#); [Abeler et al. \(2011\)](#); [Morgul and Ozbay \(2015\)](#); [Thakral and Tô \(2017\)](#); [Cerulli-Harms et al. \(2019\)](#); [Frechette et al. \(2019\)](#).

⁷ In principle ride-sharing platforms such as Uber and Lyft can use similar approach to increase drivers labor supply, for example by using information on user’s tipping behavior to match riders with drivers.

⁸ [Anderson and Green \(2018\)](#) show that on average a person reaches their personal best twice every 15 years.

collection and presents descriptive results. In Section 3, we introduce the structural model and the identification strategy. In Section 4 we show the results of the structural estimation and counterfactual analysis. In Section 5 we assess the validity of our structural modeling choices using the Cox proportional hazard model. Section 6 concludes. We provide robustness checks and supplemental material in the appendix.

2 Data

In this section, we first provide details on the data collection and information about the platform. We then provide descriptive statistics that illustrate consistent behavioral patterns.

2.1 Collection

We scraped the data from an online chess platform, chess.com, which is the most frequently visited chess website.⁹ The website has over 30 million users, and it hosts around 3 million chess games every day. Users range from amateur players to the world’s best chess players, including Magnus Carlsen, the World Chess Champion.¹⁰ This platform is free to use, and anyone can register to play against other people or a computerized opponent. The website also provides some lessons and chess puzzles.

We use the public Application Programming Interface (API) to collect data. Each observation includes information about the players and the game: usernames, their self-identified country of association, platform ratings, the time at which the game was played, which player had white pieces, the length of the game, and the final results.

2.2 Definitions

Our sample consists of “blitz” games. Blitz is a type of chess game where each player has somewhere between 3 to 15 minutes for the game. In addition, we restrict attention to games played against human players.

A *game* g is a single game with a human opponent. A collection of games ordered by time stamp, (g_1, g_2, \dots, g_n) , is called a *session* if there is no game played T minutes before g_1 or after g_n , and for any $i \in \{1, \dots, n - 1\}$, the time between g_i and g_{i+1} is less than T .¹²

⁹ Based on Alexa internet rating, www.alexa.com.

¹⁰ Magnus Carlsen, a Norwegian chess grandmaster, and chess prodigy is the highest-rated player in the world, and the highest-rated player in the history of chess.

¹¹ This table describes sub-sample of our data, because in the analysis we focus on the blitz games played against human players.

¹² For the main section of the results we set $T = 30$ minutes, we vary T to check the robustness of our results and we find no substantial differences.

Number of games	35,517,526
Number of sessions	6,063,366
Number of players	13,027
Average Session Length	4.14
Average Number of Session	348
Average Rating	1311
Rating Range	[100,2816]
Pr(Win White Pieces)	0.51
Pr(Win Black Pieces)	0.47

Table 1: Data description for our sample.¹¹

We call sessions that contain only one game ($n = 1$) *only game* (O-game). For sessions with $n \geq 2$, g_1 is the first game, g_n is the last game and any game in between the first and the last is referred to as *a middle game* ($k \in \{2, 3, \dots, n - 1\}$).

Based on the terms above, we categorize sessions into three mutually exclusive groups: (1) sessions that consist of a single game – the only game $S = (O)$; (2) session with exactly two games $S = (F, L)$; (3) sessions with more than two games $S = (F, M_1, \dots, M_m, L)$ where $m \in \{1, \dots, n - 1\}$.

Let $f_W(\cdot)$ be a function that calculates the fraction of wins in a particular type of game, for example, $f_W(L)$ is a player’s fraction of wins in the last games. In some cases, when the context is clear, instead of writing $f_W(F)$, $f_W(M)$, $f_W(L)$, $f_W(O)$, we simply write F , M , L , and O , and we mean the fraction of wins in first, middle, last, and only games, respectively.

2.3 Descriptive Results

In this section, we first establish that session-stopping behavior is history dependent. We find that a large fraction of players fall into two categories: some people consistently leave their session after they win a game, while others exit their session after a loss; we refer to these types as *win-stoppers* and *loss-stoppers*, respectively. That is, win-stoppers are more likely to continue playing after a loss than a win. Conversely, loss-stoppers are more likely to stop playing after a loss than a win. In section 3, we present a theoretical model that can account for the behavior observed in the data.

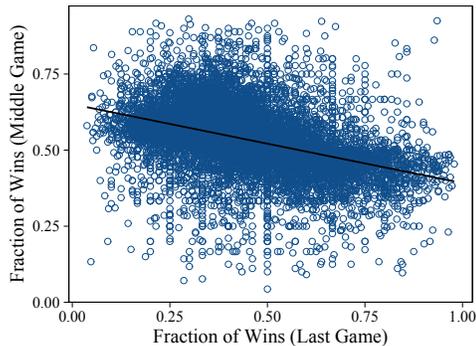
2.3.1 History Dependence

We first take all sessions in our data that lasted at least 3 games and calculate the average winning frequency in the first, middle, and last games for each player, as defined in Section 2.2. If a decision to stop the game is made randomly and stopping behavior is history independent, then the fraction of wins in the last game should be similar to the fraction of wins in any other game. Our null hypothesis is:

H_0 : *Players stopping behaviour is independent of the outcome of the last game.*

H_0 implies that the correlation between the fraction of wins in the last and middle games to be close to 1.¹³ Figure 1a presents the relationship between the fraction of wins in the last and middle scores (the solid line represents the linear regression line, where the dependent variable is the fraction of wins in the middle games and the independent variable is the fraction of wins in the last game). The correlation between the fraction of wins in the last and middle games is $-.41$ and statistically different from 1 with $p < 0.001$. Thus, at the aggregate level, the decision to stop is not random and H_0 is rejected.

(a) Fraction of wins in the Last and Middle games



(b) Density of fraction of wins

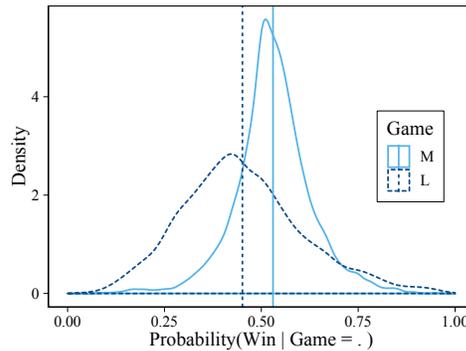


Figure 1: Fraction of wins in the Last and Middle games.

2.3.2 Behavioral Types

We rejected the null hypothesis of history independence; however, the alternative hypothesis does not identify the nature of the relationship between the outcome of the game just

¹³ We do not include first games at this stage because the data on first games is used later to check the robustness of the findings.

played and the decision to play another game. Are players more likely to end the session after a win or after a loss? We state the following two alternative hypotheses:

H_A^1 : *players are more likely to end a session after a win.*

H_A^2 : *players are more likely to end a session after a loss.*

If only one of the two alternative hypotheses is correct, we should see a skew or a shift of the distribution of the fraction of wins in the last game compared to the fraction of wins in the middle games (Figure 1b presents both distributions). The test of medians for the two distributions shows that the median fraction of wins in the last game is statistically lower than the median fraction of wins in the middle game. If we only look at this aggregate result, it supports the hypothesis that players are more likely to stop playing after a loss. However, if we take a broader look at the distribution, we see an interesting pattern. The standard deviation of the fraction of wins in the last game is twice that of the middle game. Both plots on Figure 1 informs us that there are some people with a much higher fraction of wins in the last game compared to the middle game and that there are others with a much lower fraction of wins in the last game compared to the middle game. We define players that are more likely to end the session on a loss and others who are more likely to end the session on a win in the following way:

Definition 1 *A player is behavioral type at the tolerance level of τ and she is*

- *a win-stopper if $f_W(L) > f_W(M) + \tau$,*
- *a loss-stopper if $f_W(L) < f_W(M) - \tau$,*

neutral types have $f_W(L) \in [f_W(M) - \tau, f_W(M) + \tau]$.

Using data from sessions that last 3 or more games, we classify players according to Definition 1 (see Figure 8 in the appendix for population decomposition by types using different tolerance levels). Intuitively, as we increase the tolerance level, fewer players are classified as win- or loss-stopper types. Interestingly, when we change τ the ratio of win-stoppers to loss stoppers stays stable at around 40%. At tolerance level τ of 5%, 84% of the players are classified as behavioral types, with about 30% of them being win-stoppers and a larger fraction, 70%, loss-stoppers.¹⁴

To further examine that Definition 1 captures the patterns in the data, let us examine only-games—sessions that contain only one game. Note that the sessions with only one

¹⁴ Unless specified otherwise, we use 5% tolerance level.

game have not been used in the classification; so far, we used sessions of length 3 and above. For ease of exposition, let us take Definition 1 to the extreme, where we assume that a win-stopper type **always** stops after a win and a loss-stopper type **always** stops after a loss. This extreme definition of types implies the following two observations (see Figure 2). First, the fraction of wins for win-stopper types in the only and last games must be 1. This is because if a player wins this game, she ends a session, and the game is classified as the only game. If a player loses the first game, she will start another game, making this session at least 2 games long, in which case whenever a player wins, we classify that game as the last game. Therefore the fraction of wins in the last game is also 1. Second, the fraction of wins for loss-stopper types in the only and last games must be 0 (the argument is similar to the win-stopper case). Combining these two observations leads to the following prediction.

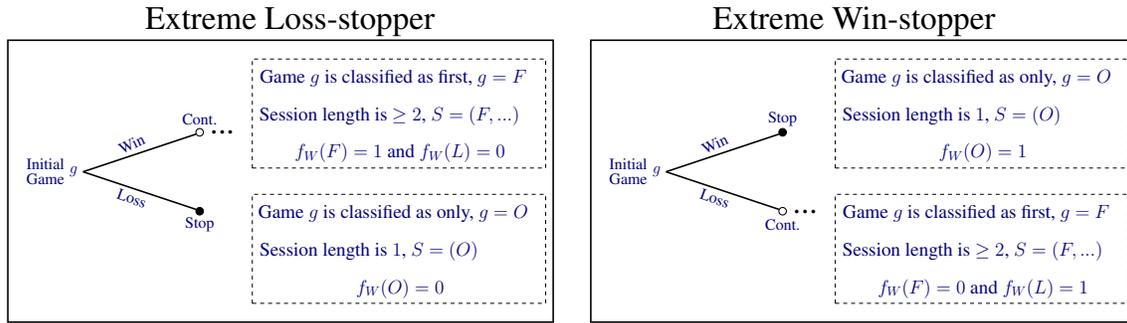


Figure 2: Extreme Behavioral Types' Actions

Prediction 1 *The correlation between the fraction of wins in the last and only game is positive.*

We calculate the fraction of wins for win-stopper and loss-stopper types in the only games as defined in Definition 1. We find that win-stoppers' fraction of wins in the only-game is two times higher than that of loss-stoppers. Figure 3 presents a scatter plot with the fraction of wins in the last-game on the x-axis and the fraction of wins in the only-game on the y-axis. A strong and significant positive relationship between the fraction of wins in the last- and only-game implies that the types who are more likely to stop playing on a win similarly have a higher fraction of wins in the only game and vice versa, as predicted in the discussion above.

So far, we used a fraction of wins in the middle, last, and only games. We have not used a fraction of wins in the first games. The diagram in Figure 2 describes what would each

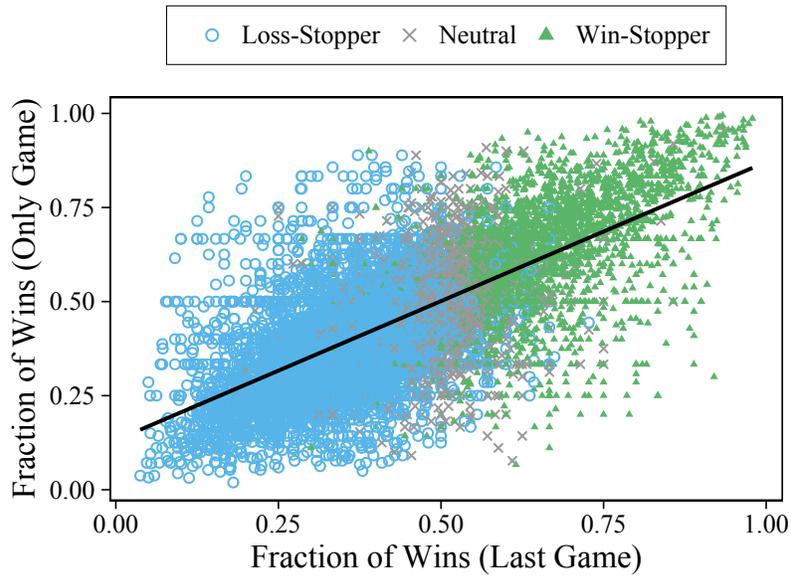


Figure 3: Fraction of wins in the Last and Only Game

type do if they won or lost the initial game. It gives a relationship between the fraction of wins in the first and only games. If the lost-stopper wins the initial game she plays another one, which makes that game classified as the first game, while if lost-stopper loses the initial game, she stops playing, which makes that game classified as an only game. Therefore lost-stopper's fraction of wins on the first game must be 1 and in the only games must be 0. Similarly, we argue that for win-stoppers, the fraction of wins in the first game must be 0, while the fraction of wins in the only game must be 1. Note that while the prediction for last- and only-game relationship is positive, the prediction for the first- and only-game relationship is negative.

Prediction 2 *The correlation between the fraction of wins in the first and only game is negative.*

Figure 4 presents the correlation matrix with p-values in parentheses. A negative relationship between the fraction of wins in the first and only games implies that the types who are more likely to stop playing on a win have a higher fraction of wins in the Only games since they would have kept playing if they had lost the initial game. Similarly, the types who are more likely to stop playing on a loss have a lower fraction of wins in the Only games.

Similar to predictions 1 and 2, we can use diagram on Figure 2 to write 4 more pre-

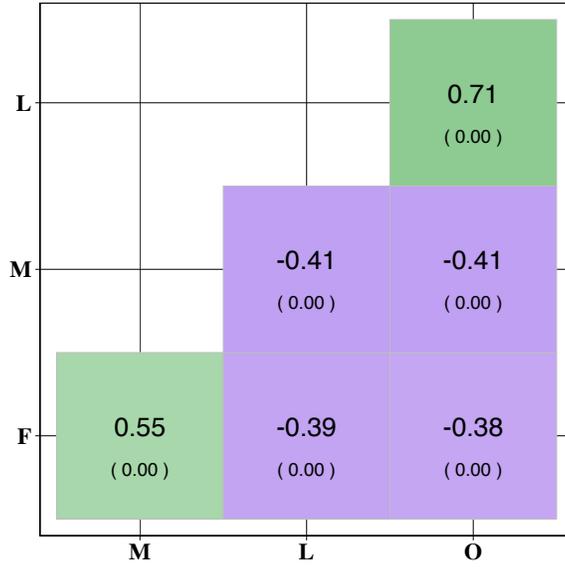


Figure 4: Correlation Matrix

dictions about relationships between the fraction of wins in the first, middle, last, and only games. All those predictions are accurately matched to the data (see Figure 4).

3 The Model

We do not take a stand on whether people end a session after a win or loss. Instead, we model different types of behavior and let the data inform us about the types. We capture the different types of people by allowing the utility from playing a new game to depend on the last game outcome and the person's type. We call a person *loss-stopper* type, if the utility from playing one more game for that person decreases after a loss compared to that after a win. Similarly, we call a person *win-stopper* type, if the utility from playing another game increases after a loss compared to that after a win. We say that a person is *neutral* type if the person's utility is not affected by the outcome of the last game.

3.1 Description

This section lays out a chess player's dynamic choice problem. A chess player is characterized by his type that consists of an element observable to the player, player's opponents and econometrician, and an element privately known to the player.

Let y be a vector of characteristics that may change over time, and are observable to the player, opponents, and econometrician (e.g. player's rating). We assume that y lives

in finite space Y . A player can be one of the following 3 types: win-stopper (θ_W), loss-stopper (θ_L), or neutral (θ_N). Let $\Theta = \{\theta_W, \theta_L, \theta_N\}$ be the set of all types and let θ be an element of this set. The player's type is fixed over time. A player's type profile at time t , (y_t, θ) , consists of player's time-variable characteristics, y_t , and fixed unobservable type, θ . We use variables without time subscripts to denote current states and 'prime' superscripts to denote the next period's state. A player's opponents' variables have subscript $-i$.

Each period, a player is facing the following decision: given the previous history of the play, the player needs to decide whether to play an additional game, or to go offline and take an outside option. Before making this decision, the player's utility from another game:

$$U(y, \theta, \chi) = u(y) + (1 - \chi)l_\theta \quad (1)$$

where y is the player's current rating, θ is the player's type, and χ is the outcome of the last game. If a player won the last game ($\chi = 1$), the utility from playing another game is $u(y)$. We can think of this term as how much the player likes playing chess independently of his type. If the player lost the last game, then her utility from playing another game depends on her type.

Definition 2 *A player is*

- i) Loss-Stopper, if $l_\theta < 0$;*
- ii) Win-Stopper, if $l_\theta > 0$;*
- iii) Neutral, if $l_\theta = 0$.*

There is an outside option, c , that every period is independently drawn from a distribution with density $f(c)$. If a player ends a session, he takes an outside option c . Otherwise, the player's utility is $U(y, \theta, \chi)$ from playing a new game and he moves to the next period, at which point the player faces exactly the same decision based on the new history of the last game (χ'). In each period a player is facing the following problem:

$$V(y, \theta, \chi, c) = \max \left\{ c, u(y) + (1 - \chi)l_\theta + \delta \sum_{\substack{\chi', y' \in \\ \{0,1\} \times Y}} p(y'|y, \chi')p(\chi'|y)V(y', \theta, \chi') \right\} \quad (2)$$

where δ is the discount factor; $p(y'|y, \chi')$ is the probability of receiving a rating y' given that the player's current rating is y and the outcome of the next game is χ' ; $p(\chi'|y)$ is the probability of the next game outcome given the current period rating y . Note that the player matching mechanism that is based on the player's own rating affects the probability of winning the next game. This is the way through which changing the matching mechanism can influence a player's decision to start a new game. Note that the law of motion of y can be directly recovered from the data (player's rating updating rule).

It is important to note that we do not take a stand on what may be motivating individuals, and focus on the corresponding reduced-form phenomenon: win-stoppers are more likely to stop playing after a win compared to a loss, and loss-stoppers are more likely to end the session after a loss compared to a win. Many papers have evaluated what psychological biases may lead to such behavior (see section B.1 for more details).¹⁵

3.2 Identification

In this section, we provide the identification of players' types, l_θ , δ , probabilities of winning, outside option distribution parameter, and matching probabilities. We start with the identification of behavioral types.¹⁶

Claim 1 *Optimal stopping rule is a threshold rule in c .*

Proof. Note that in equation (2), continuation values do not depend on the **current** realization of c . Hence, fixing the continuation values and current period utility from playing another game, the second term under the max operator is lower than outside option c , for sufficiently high c . So, we have a threshold, $\bar{c}(y, \theta, \chi)$, such that for realizations of c above this threshold, the player stops playing and takes the outside option. ■

Therefore $\bar{c}(y, \theta, \chi)$ is a threshold such that a player with type profile (y, θ) having an outcome χ in the last game ends a session if and only if the realized c is at least as large as $\bar{c}(y, \theta, \chi)$.

From (2), we have,

¹⁵ See Miller and Sanjurjo (2018b); Aharoni and Sarig (2012); Arkes (2010, 2013); Avugos et al. (2013); Cervone et al. (2014); Brown and Sauer (1993); Camerer (1989); Croson and Sundali (2005); Suetens et al. (2016); Gilovich et al. (1985); Green and Zwiebel (2017); Koehler and Conley (2003); Miller and Sanjurjo (2014, 2017a, 2015, 2018a, 2017b); Rabin and Vayanos (2010); Rao (2009b,c,a); Rinott and Bar-Hillel (2015); Sinkey and Logan (2014); Stone (2012); Stone and Arkes (2018); Sundali and Croson (n.d.); Tversky and Gilovich (1989); Wardrop (1999); Xu and Harvey (2014); Yaari and Eisenmann (2011).

¹⁶ The identification and estimation of the theory model is in the tradition of Hotz and Miller (1993). We show how we can forgo numerical dynamic programming to compute the value functions for every parameter vector, and propose an estimation procedure that is simple to implement and computationally efficient.

$$\bar{c}(y, \theta, \chi) = u(y) + (1 - \chi)l_\theta + \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi'|y)V(\theta, y', \chi') \quad (3)$$

The following proposition leads to the identification of behavioral types.

Proposition 1

- i) $\bar{c}(\theta_W, y, 0) > \bar{c}(\theta_W, y, 1)$;
- ii) $\bar{c}(\theta_L, y, 0) < \bar{c}(\theta_L, y, 1)$;
- iii) $\bar{c}(\theta_N, y, 0) = \bar{c}(\theta_N, y, 1)$.

Proof. The proof follows from equation (3). ■

Proposition 1 implies that the win-stopper types' probability of playing one more game is higher if the previous game was lost compared to when the previous game was won, and vice versa for the loss-stopper types. For the neutral types, that probability is the same no matter the history of outcomes. By Proposition 1 we can identify a behavioral type of a player from the data by simply looking at her stopping probabilities after losses and wins.

3.3 Identification of the Model Parameters

The following parametric assumption is made on the distribution of outside option, $F(c)$,

Assumption 1 $F(c)$ is an exponential distribution with parameter λ .

We now argue that under the assumption 1 and by normalizing one parameter of our choice in the model, we can identify δ , λ , l_θ and $u(\cdot)$. In the estimation, we normalize $\lambda = 1$.

Let,

$$H(y, \theta) = u(y) + \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi'|y)V(\theta, y', \chi')$$

Under the assumption 1 and from equation (3), we have that the probability of stopping and taking outside option, $h(y, \theta, \chi)$, is,

$$h(y, \theta, \chi) = e^{-\lambda(H(y, \theta) + (1 - \chi)l_\theta)} \quad (4)$$

Claim 2 $\lambda H(y, \theta)$ and λl_θ are identified for all (y, θ) .

Proof. Let's look at equation (4) for $\chi = 1$. LHS, $h(\theta, y, 1)$ is coming from the data as probability of stopping after a win. On the RHS, the second term in the power, $(1 - \chi)l_\theta = 0$, therefore we can say that $\lambda H(\theta, y)$ is identified from (4).

Now substitute $\chi = 0$ in (4). Again the LHS is coming from the data as probability of stopping after a loss $h(\theta, y, 0)$. On the RHS, first term in the power $H(\theta, y)$ is coming from the first part of this prove. Thus, λl_θ is identified from (4) too. ■

Claim 3 $\lambda V(y, \theta, \chi)$ are identified for all (y, θ, χ) .

Proof. Let's denote $\bar{c}(\theta, y, \chi) \equiv \bar{c}$. We can rewrite (2) as:

$$V(\theta, y, c, \chi) = \mathbf{1}(c > \bar{c}) * c + \mathbf{1}(c \leq \bar{c}) \left(H(\theta, y) + (1 - \chi)l_\theta \right) \quad (5)$$

Where $\mathbf{1}(\cdot)$ is an indicator function. Taking expectations of both hand sides of (5) with respect to c , gives

$$\begin{aligned} V(y, \theta, \chi) &= E(c|c > \bar{c}) + Pr(c \leq \bar{c}) \left(H(\theta, y) + (1 - \chi)l_\theta \right) \\ &= Pr(c > \bar{c}) \left(E(c) + \bar{c} \right) + Pr(c \leq \bar{c}) \left(H(\theta, y) + (1 - \chi)l_\theta \right) \\ &= Pr(c > \bar{c}) \left(\frac{1}{\lambda} + \bar{c} \right) + Pr(c \leq \bar{c}) \left(H(\theta, y) + (1 - \chi)l_\theta \right) \\ &= Pr(c > \bar{c}) \left(\frac{1}{\lambda} + \bar{c} - H(\theta, y) - (1 - \chi)l_\theta \right) + H(\theta, y) + (1 - \chi)l_\theta \end{aligned} \quad (6)$$

multiplying both hand sides by λ and substituting $\bar{c}(y, \theta, \chi)$ from (3), we get,¹⁷

$$\lambda V(y, \theta, \chi) = e^{-\lambda[H(y, \theta) + (1 - \chi)l_\theta]} + \lambda H(y, \theta) + (1 - \chi)\lambda l_\theta \quad (7)$$

Claim 2 and expression (7) imply that $\lambda V(y, \theta, \chi)$ are identified for all (y, θ, χ) . ■

Claim 4 δ and $\lambda u(y)$ are identified.

Proof.

We can consider the difference $\lambda(H(y, \theta) - H(\theta', y))$ for some $\theta \neq \theta'$. This gives us,

$$\delta = \frac{\lambda(H(y, \theta) - H(\theta', y))}{\lambda(\sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi') p(\chi'|y) (V(\theta, y', \chi') - V(\theta', y', \chi')))}$$

¹⁷ Notice that $Pr(c \leq \bar{c}) = 1 - Pr(c > \bar{c}) = 1 - e^{-\lambda[H(y, \theta) + (1 - \chi)l_\theta]}$

By claims 2 and 3, numerator and denominator are identified in the above equation. We identify $\lambda u(y)$ from,

$$\lambda u(y) = \lambda H(y, \theta) - \lambda \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi') p(\chi', y) V(\theta, y', \chi')$$

■

Finally, we can normalize all the parameters and value functions by λ . This completes the identification of the parameters of the model.

3.4 Sanity Check of the Model Classification

In this paper we have two distinct definitions of behavioral types (Definitions 1 and 2). The two definitions are intuitively related, but these two methods of identifying behavioral types do not have to overlap at all. That is, given some data, a player could be identified as win-stopper according to the model but be classified as loss-stopper according to the last-game and middle-game definition. For example, suppose a player's complete playing history contains the following set of 3 sessions:

$$\{WWWW, WLW, WLL\},$$

Let us look at all the wins and calculate the stopping probability after a win. We have $\Pr(\text{Stop}|\text{Win}) = 2/7$. Now, calculate the stopping probability after a loss, $\Pr(\text{Stop}|\text{Loss}) = 1/3$. Since probability to stop playing is higher after a loss than after a win, $\Pr(\text{Stop}|\text{Loss}) > \Pr(\text{Stop}|\text{Win})$, our model would qualify the player as loss-stopper. However, according to our definition through the last game of the session, the player's behavior corresponds to win-stopper type, because the last game is won more often than middle games.

This example demonstrates that while the model and our behavioral type definitions are intuitively related, one does not imply the other. This fact strengthens any relationship we find between the two classifications, highlighting the consistency of our intuition with the proposed theory (the coincidence of identified types from two different definitions are presented in section 4.3).

4 Structural Estimation and Counterfactual Analysis

In this section, we first introduce the structural estimation results. Then we present the counterfactual analysis results that highlight how a market designer can change how much time a player spends on the platform. Finally, we compare the types identified by the model to the initial classification from Section 2.3 and show the time stability of the model types.

To better understand the counterfactual analysis, we begin by describing the rating system on the platform. Once a player signs up for the chess.com, he gets an initial rating (1200 points).¹⁸ Rating changes after every game based on the outcome of the game. Intuitively, the rating goes up after a win and goes down after a loss. Thus, a player’s current rating reflects her current expertise in chess: higher the rating, better the player. We recover the rating updating rules from the data. To estimate parameters structurally, we divide the rating range into the grids of 20 since the rating has a wide range ([100, 2798]).¹⁹ Since we have very few observations where the rating is below 800 or above 2200, we put all the ratings below 800 as the first rating grid and above 2200 as the last rating grid (last grid is 71). The rest of the ratings are divided into 20 point intervals.

4.1 Structural Estimates

The main parameter that we estimate are l_θ for $\theta \in \{\theta_W, \theta_L, \theta_N\}$. The estimates are presented in Table 2. We sub-sample the data from the entire sample 300 times with replacement and find that our estimates are very stable.

Parameter	Mean	Std	[min, max]
l_{θ_W}	0.810	0.009	[0.776, 0.838]
l_{θ_N}	-0.067	0.004	[-0.077, 0.057]
l_{θ_L}	-0.698	0.004	[-0.706, -0.686]

Table 2: Bootstrapped values for l_θ

Table 2 shows that for a win-stopper type, utility from playing another game increases by 0.810 after a loss compared to a win. The effect is opposite for loss-stopper types, loss in the last game decreases utility from playing another game by 0.696 compared to a win. Intuitively for neutral types, last game result has only a minor effect on utility from playing.

¹⁸ Rating is an integer.

¹⁹ For the counterfactual analysis, we change the grids, more details are provided in the counterfactual section. Changing the grid size or the number of grids does not qualitatively affect any of our results.

4.2 Counterfactual Analysis

Can the market designer leverage information on behavioral types to increase the expected time spent by players on the platform? To answer this question, we need to know what can be controlled by the market designer. In our setting, the player-to-player matching algorithm is controlled by the market designer. Therefore we need to know what is the current matching algorithm, which can be recovered directly from the data. The platform has a simple matching rule. Two players with closer ratings are matched into pairs with higher probability. In other words, the platform only uses the players' ratings to decide who plays with whom.²⁰ In our counterfactual exercise, we allow the platform to choose from matching mechanisms that can be contingent on players' behavioral types in addition to the ratings.

Before presenting counterfactual results, we need to take a little detour to explain the data that we use for counterfactual analysis. In the last subsection, we presented estimates for the entire data, but for counterfactual analyses, we focus only on sub-sample. The reason is that we want players to be homogeneous on their ratings. In other words, we want to consider players who have similar ratings to avoid matching players, one of whom has a very high rating while the other player has a very low rating. Therefore, we consider only players who have an average yearly rating from 900 to 1300. Then we divide rating space [800, 1400] into 102 grids, each grid size is 6 points.

We re-estimated the model and find that even in the restricted data, the parameter estimates of l_θ are similar to what we had for the entire data (Table 3). To check the stability, we bootstrap the data again 300 times.

Parameter	Mean	Std	[min, max]
l_{θ_W}	0.718	0.005	[0.711, 0.726]
l_{θ_N}	-0.047	0.002	[-0.050, 0.045]
l_{θ_L}	-0.613	0.002	[-0.615, -0.609]

Table 3: Bootstrapped values for l_θ with restricted data

The goal of this section is to provide a matching algorithm that is better than the current one. Ideally, we would find an optimal matching algorithm among the class of algorithms that accounts for rating as well as players' types. However, due to the high dimensionality

²⁰ Obviously we assume that both players who are matched are on the platform and requested a particular type of chess game, for example, 3 minute blitz game.

of the problem (we have $3n(n - 1)$ variables to optimize, where 102 is the number of rating grids), our current computational capabilities do not allow us to find the best such algorithm. Instead, we answer the following question: at least, by how much can the platform increase the expected time spent on the platform? To answer this question, we consider optimization over a smaller class of matching algorithms and find that the platform can increase the time spent by the user theoretically by at least 62%.

There is a concern with this algorithm. The algorithm matches players with very low ratings to players with a very high rating. Such matching is not desirable for the players, because a player would not constantly play with either very low or very high rating players. Therefore we consider the matching algorithm, which does not allow for such radical matching pairs. Intuitively, such an algorithm results in less improvement – a 5.44% increase in user spent time. To put this number in perspective, let us look at simple statistics. In the sub-sample that we consider for counterfactual analysis, an average player plays 348 sessions a year, and an average session lasts about 4.14 games. Since we consider only blitz games and average blitz games in our sample lasts for 8 minutes and 12 seconds, 5.44% improvement results in about 87 more games per year (on average 11 hours and 54 minutes more time spend on the platform per year per player).

Figure 5 presents the resulting matching probabilities for four different rating grids. Green (solid) distribution represents current matching probabilities, while orange (dashed) and blue (dotted-dashed) distributions are new (improved) matching probability distributions respectively for loss-stopper and win-stopper types.

Figure 5 shows matching for 4 different grids. We chose middle grids because, for them, we have both higher as well as lower-rated possible opponents. As we explained throughout the text, if we match Win-Stoppers with relatively higher rated players, we decrease the probability that they win, so we increase the chance of playing one more. For Loss-Stoppers argument is reversed. As figure 5 shows, Loss-Stoppers should be matched with lower-rated players so that they have a higher chance of winning and continuing playing. Since the neutral types do not care about the last game result we can use them to clear the market. The matching algorithm explained above and presented in figure 5 improves average session length by 5.44%.

4.3 Consistency of Behavioral Types

We have identified each player using two classifications that are independent of each other; now, let us look at how they pair. We take all the players, and in Figure 6a we put a plus

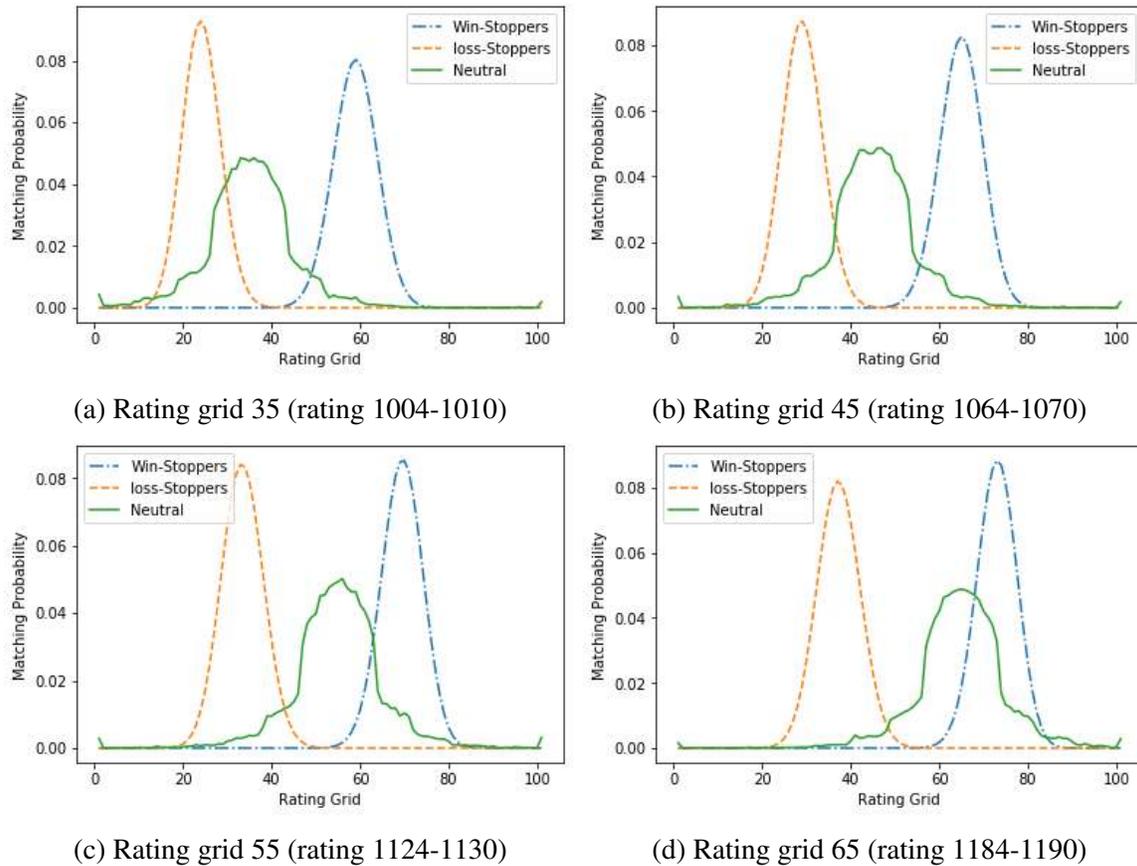


Figure 5: Matching probabilities for four different rating grids

sign if the model identifies a win-stopper player as win-stopper, loss-stopper as loss-stopper and non-behavioral as neutral. For 81% of the players in the data, the two classifications match. It provides us with strong evidence that supports our model and the claim that the game outcome affects the utility of the next game.

Figure 6b presents a transition matrix from behavioral types to model types. We observe a large mass on the diagonal, meaning that the two classifications are consistent. For example, 96% of win-stopper types identified by the model were identified as win-stopper types identified by Definition 1. However, there are mismatches; for example, some non-behavioral types are classified as behavioral types by the model and vice versa. Notice that the cases in which a win-stopper (loss-stopper) is identified as loss-stopper (win-stopper) by the model account happen only 1% of the time.

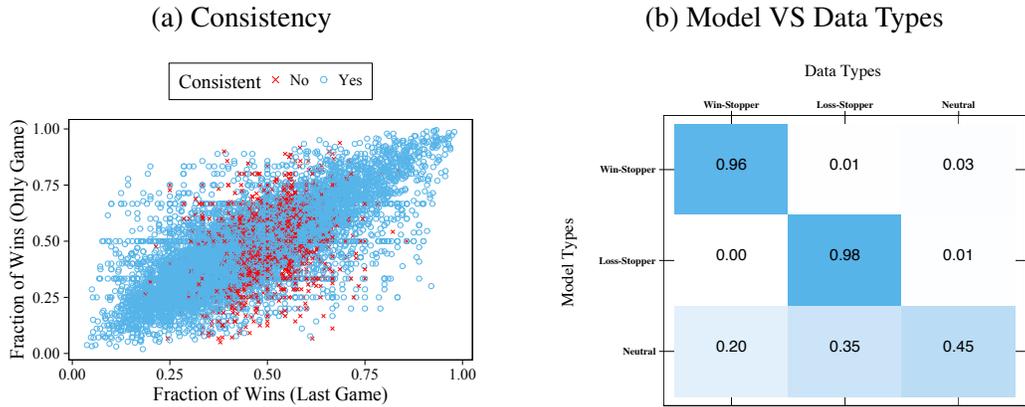


Figure 6: Model Identified Types and Consistency

4.4 Time Stability of Behavioral Types

We use data for 2017 for the estimation. However, we also have data for the same people for 2018, which we keep as a holdout sample to check the time stability of behavioral types. Figure 7 shows the relationship between these two years. First, we created individual variable break probability difference as the difference between the fraction of times a player took a break after a win and after a loss for both years. Figure 7a shows that there is a positive correlation between those two years. People who have a higher fraction of breaks after a win then after a loss in 2017, also have very similar break probability difference in 2018.

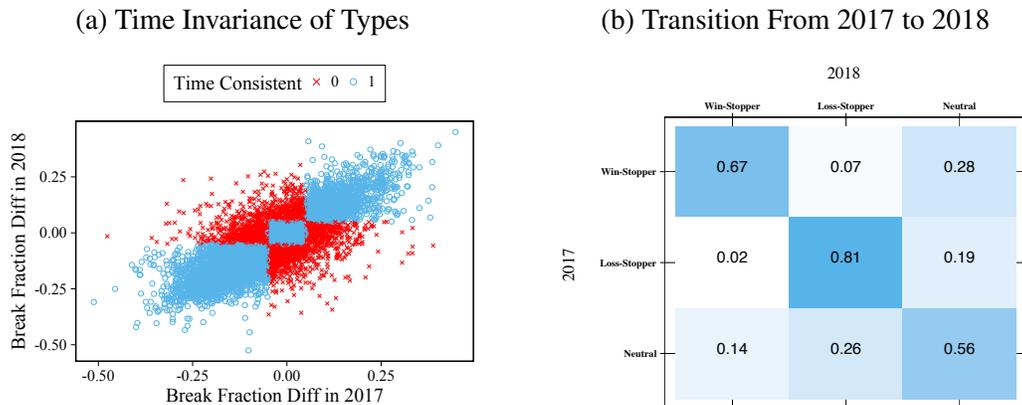


Figure 7: Time Invariance of Types (2017 to 2018)

As Figure 7b shows, the major difference that we have is among natural types, which

is predictable. The way we define natural types depends on the threshold level. Among natural types, one’s who change the type are on the threshold (it is evident from Figure 7a).

5 Validity of the Structural Modeling Choices

To show that only statistically significant and relevant factor that affects players decision to end a session is the last game result, we estimate the Cox Proportional Hazard (CPH) model. The purpose is to evaluate the effect of a number of factors on survival.²¹ Survival analysis allows us to examine how specified factors influence the session stopping rate. Such a rate is referred to as hazard rate, and the examined factors are called covariates.

We use Cox’s proportional hazard model with time-dependent covariates. We proceed by first estimating the model using aggregate data and then, later on, we estimate the model for each player separately.²² The general description of the model is as follows:

$$h_j(t, x_j(t)) = h_0(t) \exp \{x_j(t)' \beta\} \quad (8)$$

LHS of the equation 8 represents risk that game j with characteristics $x_j(t)$ is the last game of the session (session terminates after that games). RHS is comprised of two components: baseline risk and relative risk. The baseline risk, $h_0(t)$, represents the risk that a game will be the last game in a session when all the covariates equal to zero, $x_j(t) = \mathbf{0}$. The relative risk, $\exp \{x_j(t)' \beta\}$, is a proportionate increase or reduction in risk associated with the set of characteristics $x_j(t)$.

Let us look at the CPH results when we pool the data and simply look at the effect of the last game on the decision to stop a session. The estimation results are presented in Table 4.

Covariate	coef	exp(coef)	p value
Score	-0.23	0.80	< 0.005

Table 4: CPH without type heterogeneity

The variable *Score* takes value 1 if a player wins the last game and 0 otherwise. Interpreting the result of the analysis is easier using the third column ($\exp(\text{coef})$), where $\exp(\text{coef})=1$ implies whether the last game was won or lost has no effect on the decision to stop the session. The value of $\exp(\text{coef})$ is .8, and it shows that a player is 20% less likely

²¹ Survival in our setting means “not ending” a session

²² Appendix B shows the results of individual estimates for the CPH model. We also explain how the type identification using the CPH model coincides with our structural model type identification.

to stop playing after a win than after a loss. Recall that this behavior is observed among loss-stopper types, but not among the win-stopper types who are more likely to continue playing after a loss. This result is hiding an important heterogeneity that we are aware of based on the analysis in the previous sections of the paper. The estimation does not take into account any differences between players and assumes there is a universal effect of the last game outcome for all players. The negative relationship that we observe is because we have more loss-stopper types than win-stopper types in the data.

Now we introduce behavioral type heterogeneity in players according to our model estimation in Section 3. For the ease of exposition of the results, we assume that there are no neutral types, and we have only two types of players: loss-stopper and win-stopper types. We include the player’s type and interaction of type and the last game result to the covariates and re-estimated the CPH model. The results are presented in table 5.

Covariate	coef	exp(coef)	p value
Score	-0.54	0.58	< 0.005
Type	-0.55	0.58	< 0.005
Type \times Score	1.01	2.76	< 0.005

Table 5: CPH with type heterogeneity

As mentioned earlier, the variable *Score* takes value 1 if a player wins the last game and 0 otherwise. The variable *Type* equals to 1 for win-stopper types and it is 0 for loss-stopper types. Therefore, the baseline in the estimation is loss-stopper type losing in the last game. Table 5 shows that for a win-stopper type (type=1), the hazard rate is higher after a win than after a loss.²³ In other words, a chance that a win-stopper type ends a session after a loss is lower than after a win. That is what we expected. For loss-stopper types, the relationship is reversed. A win in the last game decreases the hazard rate compared to a loss (baseline) by $100-58=42\%$.

Up until this point, we have only focused on the effect of win/loss history for just one game. Now, we examine whether there is an effect of more than one lag on the decision to stop a session. We estimated the CPH model as before, but now we add lag 1 score and interaction of lag 1 score and player’s type.²⁴ Table 6 presents the results of the estimation.

Adding the outcome of the game previous to the last game has little effect on the esti-

²³ Hazard rate is lower by $-0.54-0.55+1.01=-0.08$ for a win-stopper type after a win compared to baseline and lower by -0.55 after a loss. Therefore loss decreases the hazard rate much more than a win.

²⁴ Lag 1 game is the game before the last game.

Covariate	coef	exp(coef)	p value
Score	-0.62	0.54	< 0.005
Type	-0.65	0.52	< 0.005
Type × Score	1.13	3.08	< 0.005
Lag 1 score	0.04	1.04	< 0.005
Type × Lag 1 score	-0.04	0.96	< 0.005

Table 6: CPH with type heterogeneity and Lag 1 game

mates. The last game outcomes have effects similar to what we observed in Table 5. The lagged values have statistically significant effects, but the magnitude of the effect is much smaller than the effect of the last game outcome. Note that the sign of the effect of the lagged variable is predicted using the arguments presented in Section 2.3.2.

There is one other factor that might have an effect on stopping decision. If a player's stopping rule is to end a session once she reaches a higher rating compared to what she started with, then the rating difference since the first game of a session should be significant. To test this hypothesis, we included rating change since the start of the session in the CPH and re-estimated the model.

Covariate	coef	exp(coef)	p value
Score	-0.61	0.54	< 0.005
Type	-0.67	0.51	< 0.005
Type × Score	1.10	3.01	< 0.005
Rating Change	0.00	1.00	< 0.005
Type × Rating Change	-0.00	1.00	< 0.005

Table 7: CPH with type heterogeneity and rating change since the start of the session

Table 7 shows that the change of the rating since the session started is not an important variable for deciding to end a session or not, neither is interaction term between the type of the player and rating change. It is important to note that change in rating after every game is in the interval of $[-12, 12]$, the coefficient for rating change is 0.002, and for rating change and interaction term is -0.001; therefore, the effect of rating change is not essential for the stopping decision.

6 Conclusion

In this paper, we investigate stopping behavior in an environment free of monetary incentives and identify factors that determine how people make a stopping decision. We identify two types of people: win-stoppers and loss-stoppers. Win-stoppers are more likely to stop playing after a win, while Loss-stoppers are more likely to stop playing after a loss. With conservative parameter values, we classify about 75% of players as behavioral types, one-third of which are win-stoppers, and the rest loss-stoppers.

We develop a dynamic discrete choice model in which the agent may have non-separable preferences over the stochastic outcomes of their actions. Our structural model allows for the future game utility to depend on the current game result and is able to capture the heterogeneity in stopping behaviour. We use our structural model to test alternative market designs. Our counterfactual analysis show that using type identification in matching algorithm can increase session length by 5.44%.

Although the industry for online games is huge, little is known about the determinants of length of play. In this paper, we document strong behavioral forces that drive players' stopping decisions. The results in the paper are not limited to a chess game, and the results can be applied to online games more broadly. Our approach can be used in other settings as well. For example ride-sharing platforms such as Uber and Lyft can use similar approach to increase drivers labor supply, by using information on user's tipping behavior to match riders with drivers.

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Appendices

A Robustness

A.1 Robustness of Tolerance Threshold

Figure 8 presents the players' population decomposition by types as we vary τ from 0 to .2. We see that behavioral types are robust to changing the allowed tolerance.

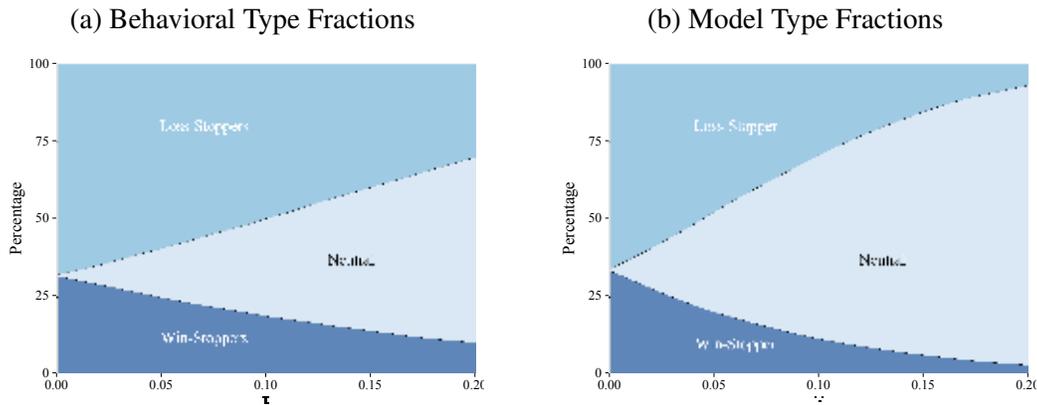


Figure 8: Type Decomposition

Similarly, changes in κ , also have consistent effect on model type decomposition, Figure 8b.

A.2 Rating and Time Invariance of Behavioral Types

In this section, we examine whether a person's type changes over time and whether the rating in the chess game is correlated with the type classification. To study these questions we take two approaches: (i) we use existing data, and (ii) we collect additional data.

Rating Figure 9a presents the break down of types over different rating categories. All three classification types are represented at every rating level. Moreover, the ratio of types for non-extreme rating levels are similar. Figure 9b presents the the distribution of the rating by type classification and we find that the distributions are practically the same. Kolmogorov-Smirnov test between win and loss-stoppers finds no difference with $p = 0.4$. Non-behavioral types have on average lower rating than behavioral types with $p < 0.01$.

To further examine and look for any possible differences in rating and type classification we collect data for a special subset of chess players that have rating in top 2000. We do not find any qualitative difference in the distribution of behavioral type or in the ratio of win and loss-stoppers. These result suggest that the rating in the chess game is not related to the player’s type.

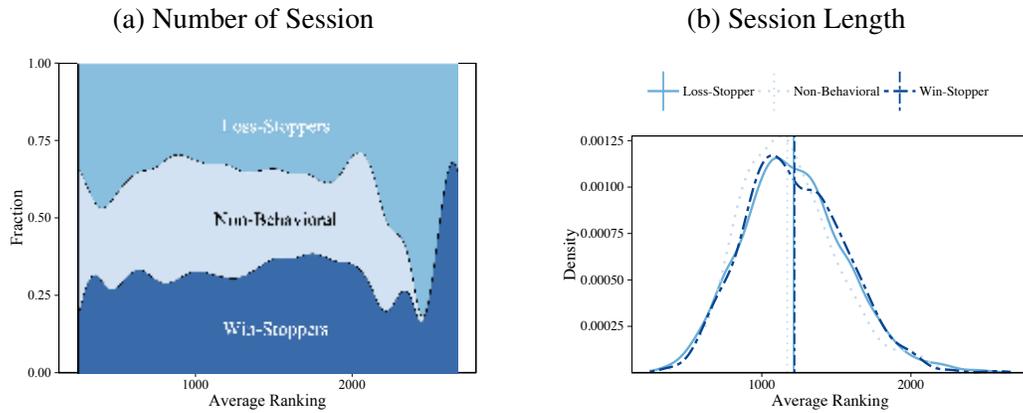


Figure 9: Ability and Behavioral Types

Time To study whether the behavioral friction that we find in this paper is an inherent trait that does not change over time we collect and analyze data from year 2018 and compare to the results that are based on the year 2017. We use the data for same players in 2018 and we find that there is 85% type match between this two consecutive years, highlighting that the individual patterns we observe do not change over time.

A.3 Number of sessions and session length

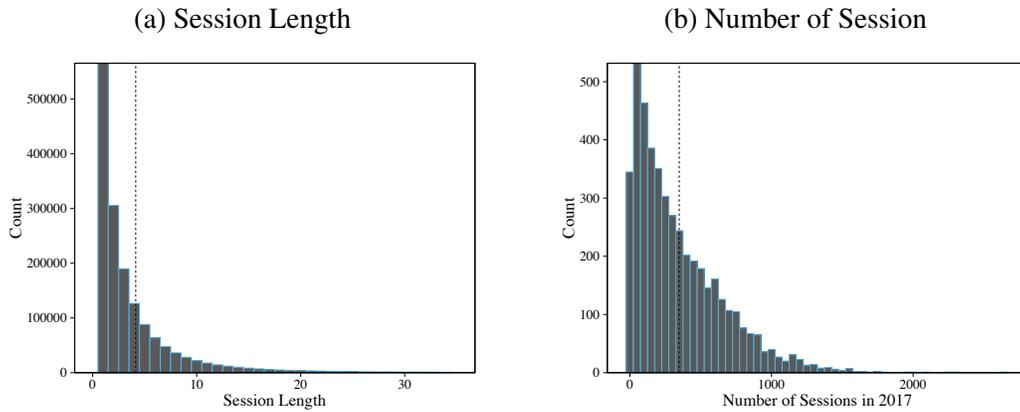


Figure 10: Session length and number of sessions per player

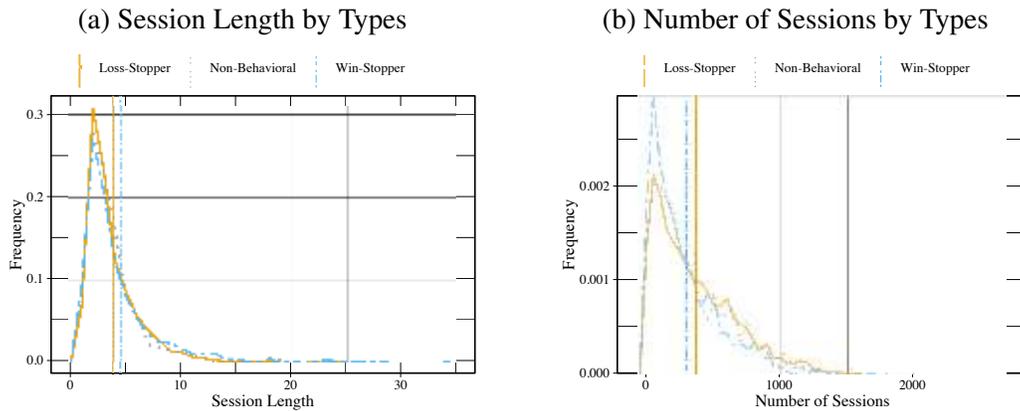


Figure 11: Session Statistics by Types

A.4 Crowding Out

Our counterfactual shows that taking into account players behavioral type for matching algorithm can increase average number of games played during the session. One might think of several crowding out arguments that increase of session length might have. Even though we can never know exact answer to such questions without RCT, in this subsection we provide evidence that such an effect is not likely.

More games does not guarantee more time

The goal of counterfactual exercise was to increase number of games during the session, but the goal of market designer (platform) is to increase time spend on the platform. To address this issue we calculated correlation for every individual between minutes spend on the platform during a session and number of games played in the same session.

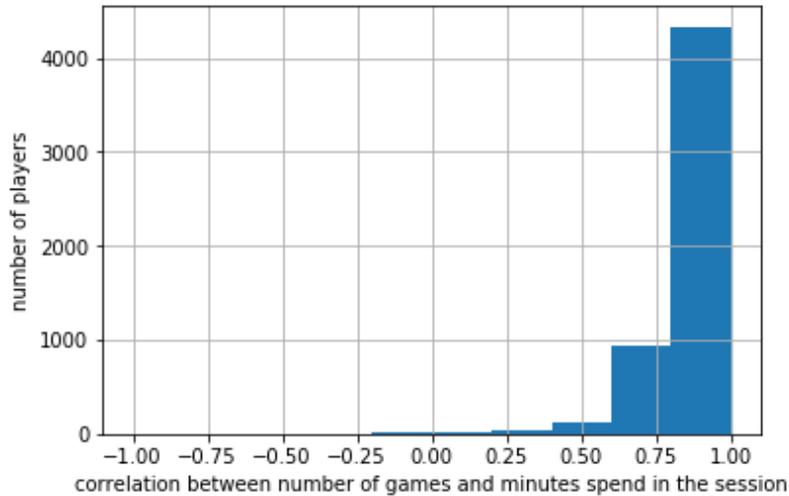


Figure 12: Correlation between minutes spend for a session and number of games

Figure 12 shows that correlation is very high. Median correlation between minutes and games during the session is 0.98 across players.

Asymmetric matching can decrease playing time

Second issue that one might worry about is that asymmetric matching can cause fast games, in the sense that strong players can win with a weak players very fast. To show that that will not be an issue we calculated correlation between rating difference and minutes spend on a game. As we explained in the text, rating is a measure of how good a player is in chess. Higher rating means better player. Rating difference gives as measure of how much better one player is compared to another.

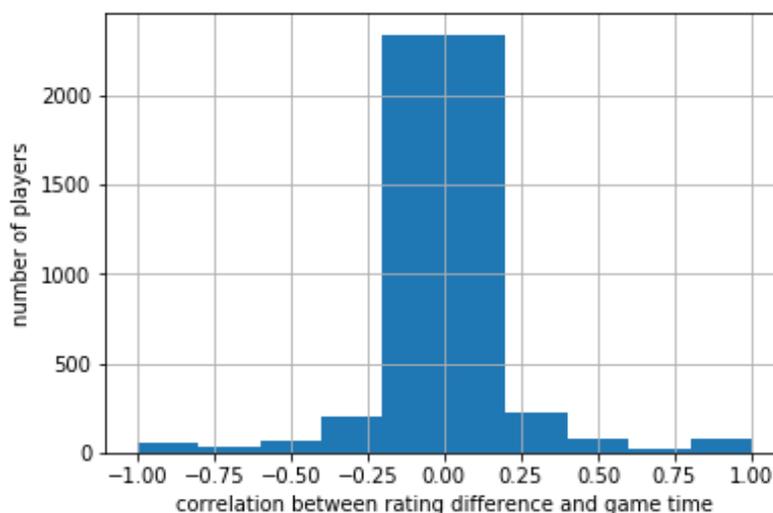


Figure 13: Correlation between minutes spend for a game and rating difference

Figure 13 shows correlation between how much better your opponent is and how much time the game lasts is almost zero for most of the players.

One long session can cause next session to be short

To answer this concern first let us present statistics about number of session per day. Figure 14 shows that median player plays two sessions a day. One might worry that if a first session increases during the day, it can decrease next session length. We find that correlation between number of sessions played during a day and average length of a session is 0.0002.

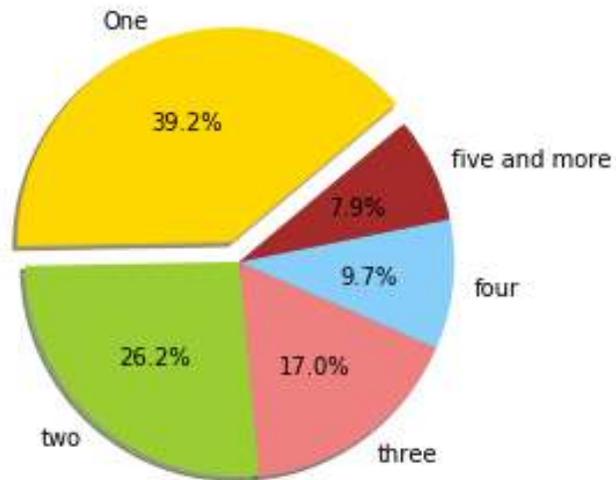


Figure 14: Number of sessions a day

We also checked auto-correlation and find that one session length does not tell anything about the next session length.

Players adjust game type based on the time they have played

The last issue that we address here is about changing the type of the game. It might be that person who started a session with 5 minutes blitz games can play shorter last game (for example 3 minute game) because she has only certain time allocated to play on the platform. If that is the case we should see that people change game types during the session.

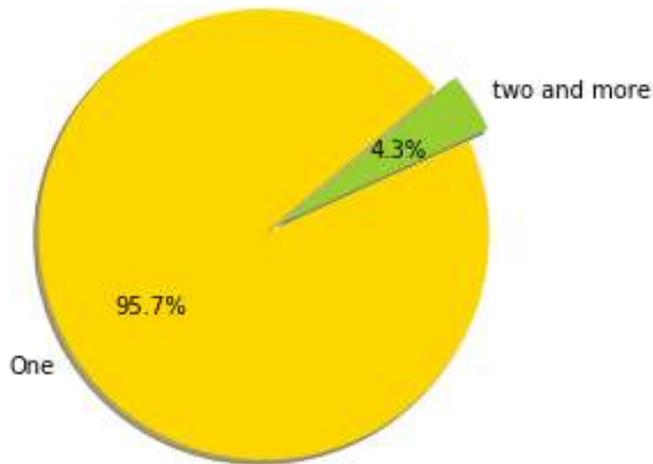


Figure 15: Homogeneity of the sessions

Figure 15 presents the evidence against the argument from above. We see that 96% of sessions are homogeneous in the sense of game type. This includes not only change if game type in the last game but during any other time. Which makes our argument even stronger that players do not choose last game type based on the remaining time that they have allocated for plying chess.

B Estimating CPH Model for Each Individual Separately

In our data set, an average player plays 348 session a year and an average session lasts 4 games. Therefore, we have sufficient data to estimate CPH model for each individual player. By examining the individual estimates we can identify win-stopper, loss-stopper and neutral types by looking at individual coefficients and the corresponding p -values. Since we already identified types using two different methods (one comparing winning fractions in the last and non-last games and other from the structural model), this is another robustness check of consistency of types over different methods.

Figure 16 presents the distribution of individual coefficients for the last game outcome variable. The figure highlights heterogeneity among the players.

Let us look at the differences in the hazard rate coefficient distributing of CPH model by behavioral types using the structural model estimation. Figure 17 shows the coefficient distributions for loss-discourage and win-stopper type separately. As we see win-stopper types have positive coefficients (Figure 17b), which means that win in the last game increases the chance that the player will end a session. For loss-stopper types (Figure 17a),

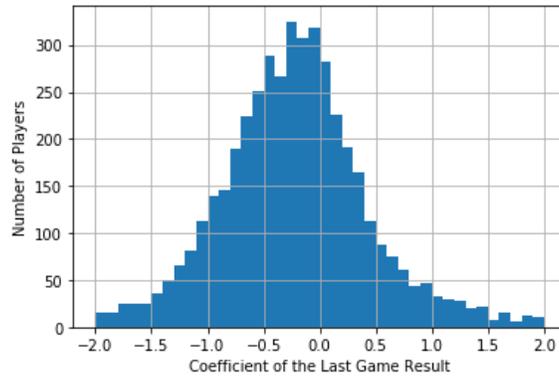


Figure 16: Individual coefficient distribution

the effect is opposite, win in the last game decreases the hazard of ending a session.

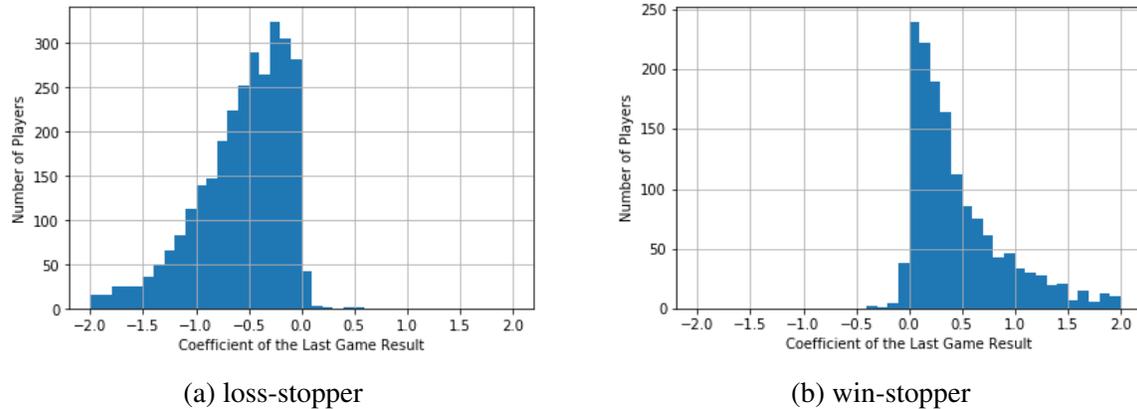


Figure 17: Individual coefficient distributions for each behavioral type

B.1 Theoretical Discussion

What would explain the patterns of behavior that we find in the data? Can reference dependence, fatigue, gambler’s fallacy, or hot hand fallacy explain the observed patterns? Let us start with reference dependence. One’s personal best rating could act as a reference point for a player: a player ends a session whenever she sets her new personal best rating, and she plays longer otherwise. Reference dependence could only predict one type of behavior – stopping the session on a win. However, loss-stoppers can not be explained by reference-dependence unless we assume players have goals to lose a certain number of games every time they play. While theoretically possible, we have neither intuitive nor statistical support for such targets.

Another theory that we discuss is fatigue. Suppose that as a player keeps playing games, she gets fatigued over time; hence, fatigue would lead to worsened play over time. While lower last game scores are observed among loss-stoppers, it is opposite for win-stoppers, who have a much higher score in the last game. Moreover, even for loss-stoppers, we do not find any support of the fact that players' performance worsens as they play more games during a session.²⁵

The last two theories that we consider are the gambler's fallacy and the hot hand fallacy. We call them belief-based explanations because both of them are based on players' beliefs about their future performance. The first one – the gambler's fallacy – implies the maturity of chances, if something happens more frequently than normal during a given period, it will happen less frequently in the future. The gambler's fallacy suggests that if a player wins a few games in a row, then a player may think that it is less likely that she wins again, and therefore, stops on a win. Similar logic applies if a player has lost a few games in a row, then she may think that she is more likely to win, therefore assuming person likes winning more than losing, she should stay and not end a session on a loss. While gambler's fallacy could explain some patterns found among win-stoppers, it goes against the behavior of loss-stoppers.²⁶

The second belief based theory is the hot hand fallacy. Some athletes (as well as their fans) believe that if they succeed several times in a row, they have a "hot hand", i.e. higher chances of success in the next attempt too. It implies that if a player believes in the hot hand, that player should keep playing in case of win and stop after lose since it indicates that she does not have a hot hand anymore. Such behavior can predict the last game result of the loss-stoppers, however it goes against the behaviour of win-stoppers.

To sum up, all the existing theories can explain one part of the story but is silent about the other. Win-stoppers might be the players who believe in gamblers fallacy and loss-stoppers be the one's who believe in hot hand fallacy. It can be that the entire population is a mixture of people who believe in different theories or principles. In this paper, we do not take a stand about underline psychological forces that motivate such behavior. Instead, we propose a theory that explains the actions found in the data and accounts for heterogene-

²⁵ We ordered games for every session and find that fraction of wins is not affected by the order number of the game. We also conducted logit estimation to see if winning probability is affected by the order number of the game. We find that order number is not a statistically significant variable.

²⁶ In addition, if a player believes in the gambler's fallacy, that player's stopping decision should depend not only on the last game but on the game previous to last (lag 1 game). We show in Section 5 that the effect of lag 1 game result is weak compared to the last game result.

ity. Therefore, our model can accommodate win-stoppers, loss-stoppers, and neutral types together in the same model.