# Forgetful Consumers and Consumption Tracking

Ying Bao* Peter Landry† Mengze Shi‡

October 4, 2022

**Abstract**

We study the market consequences of advances in consumption tracking technologies — such as mobile banking apps that help consumers monitor their spending and avoid overdrawn accounts — using a two-period consumption model. In the model, consumers pay a penalty fee if they consume in both periods. In the second period, consumers may be forgetful of their first-period consumption, though the use of consumption tracking can remind them. According to our analysis, the availability of consumption tracking often helps consumers at the expense of the firm; such benefits may be *direct*, where consumers make use of the technology to avoid penalty fees, or *indirect*, where the mere availability of consumption tracking forces the firm to lower its penalty fee. If consumers are *partially* sophisticated regarding their forgetfulness, however, the availability of consumption tracking may instill a false sense of security in that consumers *expect* to use consumption tracking to avoid penalty fees, but ultimately decide not to bother, making them especially susceptible to penalty fees. In some cases, the availability of consumption tracking may actually compel a firm to impose a penalty fee that would not otherwise be viable, leading to higher profits and lower consumer surplus.

**Keywords:** Forgetfulness, contract design, technology adoption, bounded rationality

* University of Illinois at Urbana and Champiagn, Gies College of Business, 1206 S Sixth Street, Champaign, IL, USA Email: ybao@illinois.edu.

† Department of Management, University of Toronto, Mississauga, and Rotman School of Management, University of Toronto, St. George; 3359 Mississauga Road, Mississauga, ON, Canada L5L 1C6. Email: Peter.Landry@rotman.utoronto.ca.

‡ University of Toronto, Rotman School of Management, 105 St George St, Toronto, ON, Canada M5S 3E6. Email: Mengze.Shi@rotman.utoronto.ca.

# 1 Introduction

Penalty fees are prevalent in many industries. As one example, US banks collected over 15 billion dollars in penalty fees from overdrawn bank accounts in 2019 alone (CFPB, 2021). Similarly, cellular service contracts often impose penalties for subscribers who exceed their monthly allotment of minutes and/or data.

Empirical research suggests that penalty fees are often accrued as a result of consumers' *forgetfulness*. In a British Financial Conduct Authority (2008) survey, only 7% of consumers who were charged penalty fees for overdrawn bank accounts expressed that "they knew it would happen but had to make a payment"; instead, most indicated that they lost track of their spending and incorrectly believed that they had enough remaining funds to avoid overdrawing their accounts.[1] Furthermore, empirical research suggests that consumers tend to *underestimate* the extent of their forgetfulness, e.g. in remembering to claim rebates (Ericson, 2011; Rodemeier, 2021) and in scheduling dental appointments (Altmann, Traxler, and Weinschenk, 2021).[2] In markets with penalty fees, consumers' lack of "sophistication" in this sense would naturally make them less cautious in their initial use of a service — and by extension, *especially* vulnerable to penalty fees.

The recent proliferation of consumption tracking technologies — such as mobile banking apps that help consumers monitor their spending and automated text alerts that warn consumers on the verge of exceeding their monthly allotment of cellular data — represents a potentially promising and significant development for consumers who struggle to avoid penalty fees. However, it remains to be seen whether (or to what extent) these technologies will deliver on their promise. Indeed, it is not entirely clear how consumption tracking technologies may impact market behavior and outcomes. For instance, will consumers actually make use of these technologies? How will service providers respond? Will penalty fees be eliminated?

[1] Along similar lines, Stango and Zinman (2014) find that asking questions about overdraft fees may act as subtle reminders that "induce the customer to monitor balances more closely," thus helping account holders avoid such fees. For additional evidence of consumer forgetfulness, see Helgeson and Beatty (1987), Dickson and Sawyer (1990), and Grubb and Osborne (2015), among many others.

[2] Such findings parallel evidence that consumers are *partially sophisticated* regarding their time-inconsistent preferences, meaning they tend to underestimate the extent of the present bias (DellaVigna and Malmendier, 2006; Acland and Levy, 2015; Augenblick and Rabin, 2019); reinforcing this link, Landry (2019) demonstrates how present bias may indeed be a manifestation of consumer forgetfulness.

This paper analytically investigates the market effects of advances in consumption tracking technologies. In our model, a firm offers a two-period service contract to a consumer market. This contract is defined by a subscription price and a penalty fee that is incurred by consumers who use the service in both periods. Importantly, consumers may be forgetful of their first-period usage by the time the second period arrives; in addition, consumers may only be *partially* sophisticated in the sense that they initially underestimate the extent of their forgetfulness. That said, consumers *might* have access to a consumption tracking technology that, if used, reminds them of their first-period usage prior to their second-period decision.

To assess the effects of advances in consumption tracking technologies, we compare the market-level predictions of our model with consumption tracking to the predictions of a benchmark model without consumption tracking. (Advances in consumption tracking technology may, in our model, also be conceived as a reduction in the cost associated with tracking one's consumption.) As our analysis illustrates, however, advances in consumption tracking technologies are not necessarily consequential; in particular, the availability of consumption tracking has no effect on equilibrium behavior if the cost (or "hassle") from using the technology is sufficiently high, if consumers have sufficiently reliable memories, and/or if consumers are sufficiently sophisticated regarding their forgetfulness.

Turning to the remaining cases in which the availability of consumption tracking *is* consequential, our analysis reveals multiple channels through which advances in consumption tracking technologies can help consumers at the expense of the firm. As one possibility, the availability of consumption tracking can compel the firm to reduce (or even eliminate) its penalty fee to prevent consumers from using the technology to avoid penalty fees; in these cases, consumers benefit from the availability of consumption tracking even though they do not actually use the technology. If consumption tracking is only available to a small segment of consumers, however, these consumers may *directly* benefit by making use of the technology to successfully avoid penalty fees. With an especially low tracking cost, however, these consumers may no longer be served in equilibrium — effectively "cancelling" their subscriptions; ironically, these unserved consumers still benefit from the availability of consumption tracking because those who are served overpay for their subscriptions and thus receive negative expected surplus (despite anticipating zero expected surplus).

The outcomes described above naturally fit with an intuition that advances in consumption tracking technologies would generally *hurt* the firm by limiting its ability to profit from penalty fees. That said, our analysis also reveals conditions under which the availability of consumption tracking can enable the firm to *increase* its profits. Namely, if consumers are moderately sophisticated and with moderate costs of tracking one's consumption, consumers may *plan* to use consumption tracking to avoid penalty fees, yet ultimately decide not to bother. In these equilibria, the availability of consumption tracking paradoxically makes consumers *more* vulnerable to penalty fees — while potentially allowing the firm to earn higher profits — by instilling a false sense of security whereby consumers expect to make use of the technology to avoid penalty fees, but ultimately fail to follow through.[3]

Besides accommodating the possibility that consumption tracking can help the firm increase its profits, equilibria in which consumers falsely expect to use consumption tracking can entail other unique properties. For instance, the firm may optimally set a positive penalty fee even though it would have set its penalty fee to zero if consumption tracking had not been available; in this way, advances in consumption tracking technologies may compel a firm to impose a *new* penalty fee that was not previously viable. In other cases, the firm may reduce its penalty fee relative to its optimal level without consumption tracking, yet still collect more total revenue from penalty fees — and earn higher total profits — as consumers become more likely to incur penalty fees based on their false expectation that they will use consumption tracking. Interestingly, however, consumers are not necessarily hurt in these cases as their overall usage of the service — and hence, the total value derived from their usage — increases, while this increase in value can outweigh the increase in consumers' total payments to the firm. This suggests that the availability of consumption tracking technologies can be mutually beneficial to consumers and the firm — though this can only happen if it gives consumers false hope that they will use consumption tracking to avoid penalty fees.

[3] This idea fits with more general notions of a "technology effect," in which consumers may be overoptimistic that a new technology will lead to desirable outcomes (Clark, Robert, and Hampton, 2016).

Although there is relatively little research investigating the market effects of advances in consumption tracking technologies, our study can still be related to multiple existing streams of research. To begin, our study relates to the growing literature on three-part tariffs — where the "third part" of the tariff may be interpreted as a per-unit penalty fee — in service contracts. Most work in this area does not address the use of penalty fees in markets with forgetful consumers (e.g. Sundararajan, 2004; Lambrecht, Seim, and Skiera, 2007; Ascarza, Lambrecht, and Vilcassim, 2012; Bagh and Bhargava, 2013; Herweg and Mierendorff, 2013; Leider and Şahin, 2014; Desai, Purohit, and Zhou, 2018; Fibich, Klein, Koenigsberg, and Muller, 2017; Guo, 2022); that said, there are some important exceptions (Grubb, 2015; Grubb and Osborne, 2015; Liu, Montgomery, and Srinivasan, 2018; Chen, Jiang, and Shah, 2022).

The most closely related of these works is Grubb (2015). As in our model, Grubb considers consumers who are forgetful (or "inattentive") towards their past consumption in markets with penalty fees. Unlike our focus on the market-level consequences of advances in consumption tracking technologies, however, Grubb addresses an entirely different research question by focusing on the effects of "bill-shock" regulations that require firms to alert consumers who are on the verge of incurring penalty fees. These alerts may indeed be implemented through technologies (such as automated text messages) that have much in common with the consumption tracking technologies considered in our paper. That said, a regulator determines whether to implement these alerts, which — as modeled by in Grubb — effectively force *all* consumers to track their consumption (at zero cost); by contrast, a segment of consumers in our model may have access to a consumption tracking technology, and those that do endogenously choose for *themselves* whether or not to incur a cost (e.g. a hassle cost) to track their consumption.

Another differentiating feature of our study is that we consider *partially* sophisticated consumers who know they are forgetful but underestimate the extent of their forgetfulness, while Grubb only considers consumers who are fully naive or fully sophisticated. This distinction may at first seem modest in comparison to the differences discussed above. However, our consideration of partially sophisticated consumers — which, as noted earlier, is empirically well-supported (see footnote 2 and surrounding discussion) — proves highly consequential in our analysis. For instance, we find that consumers must be partially sophisticated for the firm to earn higher profits as a result of advances in consumption tracking technologies. Similarly, advances in consumption tracking technologies can only lead a firm to impose a new penalty fee (that would not otherwise exist) and can only be mutually beneficial to consumers and the firm if consumers are partially sophisticated. At the heart of these unique market outcomes is the possibility that partially sophisticated consumers may *expect* to track their consumption yet decide against it when the opportunity arrives; importantly, only partially sophisticated consumers can hold such false expectations as fully sophisticated consumers would always follow through if they expect to use consumption tracking, while fully naive consumers would never expect to use consumption tracking in the first place.

Our work also relates to the empirical analyses of Grubb and Osborne (2015) and Chen et al. (2022). As in our study, these studies consider optimal contract design in markets with penalty fees. That said, these studies do not address consumers' potential forgetfulness of past consumption — not to mention the possibility that consumers may be partially sophisticated with regards to their forgetfulness (though Grubb and Osborne's representation of consumer overconfidence does resemble our representation of naivete). These studies also follow Grubb (2015) in focusing on the effects of bill-shock regulation, as opposed to our focus on the effects of advances in consumption tracking technologies. Interestingly, the counterfactual analysis of Grubb and Osborne (2015) suggests that consumers would be hurt by bill-shock regulation while the counterfactual analysis of Chen et al. (2022) suggests that consumers would be helped.

Although the market effects of advances in consumption tracking technologies are not explored, Liu et al.'s (2018) empirical analysis of banking overdraft provides many relevant insights for understanding consumers' responses to penalty fees. For instance, Liu et al.'s findings highlight the importance of tracking costs in deterring consumers from monitoring their bank accounts. The importance of tracking costs in deterring the use of consumption tracking is also a recurring theme in our analysis; as we find, the firm will often leverage this effect by strategically setting its penalty fee at a level that ensures a consumer's incentive to track their consumption is insufficient to overcome the disincentive from the tracking cost. Another noteworthy finding from Liu et al.'s study is that consumers who frequently overdraft their bank accounts are disproportionately likely to "front-load" their spending during a given pay period. This tendency is indeed captured by our model.

In addition to the (relatively few) studies that have addressed consumer forgetfulness of past consumption in markets with penalty fees, researchers in marketing and economics have modeled other forms of forgetfulness in consumer decision-making. For instance, several studies have explored markets with consumers who are forgetful of past prices (e.g. Helgeson and Beatty, 1987; Dickson and Sawyer, 1990; Chen, Iyer, and Pazgal, 2010). As another approach, others have modeled markets with consumers who may forget to consider a particular product (e.g. Sahni, 2015; Lovett and Staelin, 2016; Landry, 2022). Others, meanwhile, have instead studied consumers who are forgetful in their evaluations of products and/or brands (e.g. Mehta, Rajiv, and Srinivasan, 2004; Shapiro, 2006; Villas-Boas and Villas-Boas, 2008).

The rest of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we analyze a benchmark version of our model without consumption tracking. We then analyze the market effects of consumption tracking in Section 4. In Section 5, we elaborate on the managerial and policy implications of our study and address directions for future research.

## 2 The Model

We consider a market with a monopoly firm and a unit mass of consumers. At $t = 0$, the firm offers a service contract and consumers then decide whether or not to subscribe to the service. A subscription allows a consumer to use (consume) the firm's service in each of two consumption periods, $t = 1,2$. At each consumption period $t$, a consumer learns her current valuation of the service $v_t$, which is independently drawn from a uniform distribution between 0 and 1, and then decides whether or not to consume.[4] We let $d_t \in \{0,1\}$ denote the consumer's period $t$ consumption

[4] Here, we are following the precedents of Grubb (2015) in modeling a binary consumption choice made in two periods with temporally independent valuations. While crude, a two-period, binary-choice consumption model arguably provides the simplest possible framework that allows us to meaningfully study the market effects of advances in consumption tracking technologies with forgetful consumers. A variation of our model with time-invariant (as opposed to temporally independent) valuations is considered in Appendix A.1.

choice, $D_t \equiv \Pr[d_t = 1]$ denote the (ex-ante) probability of consuming in period $t$, and $D_{12} \equiv \Pr[d_1 = d_2 = 1]$ denote the probability of consuming in both consumption periods.

The service contract has two components. The first is a subscription price, $p$, which is paid by a consumer who subscribes to the service regardless of her usage in subsequent consumption periods. The second is a penalty fee, $\phi$, which is paid by a consumer who consumes in both consumption periods. Thus, with $\phi > 0$, we can think of a subscription as providing one unit of consumption, with a penalty for consuming in excess of this allocation. We assume (for simplicity) that all market participants are perfectly patient (i.e. no discounting) and that the marginal cost of providing the service is zero. The firm's expected profit from a consumer who subscribes to the service is then:

$$\Pi = p + D_{12} \cdot \phi. \tag{1}$$

Besides the $t = 0$ subscription decision and the $t = 1, 2$ consumption decisions, a consumer may face one additional decision. Namely, upon arriving at $t = 2$, but before learning $v_2$, a consumer with access to a consumption tracking technology decides whether to incur a *tracking cost* $k \geq 0$ — which may include any time, effort, or psychological cost of tracking one's consumption (and to the extent that it may also include a monetary cost, it is not paid to the firm) — to use this technology, where $d_\tau \in \{0, 1\}$ denotes whether a given consumer tracks her $t = 1$ consumption and $D_\tau \equiv \Pr[d_\tau = 1]$ denotes the probability that the consumer uses consumption tracking. The use of consumption tracking allows a consumer to remember, with certainty, whether or not she consumed at $t = 1$. Without consumption tracking, however, a consumer who consumed at $t = 1$ believes, when $t = 2$ arrives, that there is a probability $\alpha \in (0, 1)$ that consumption did *not* occur at $t = 1$. Thus, a higher value of $\alpha$ means the consumer is more *forgetful* in the sense of assigning a higher probability to the incorrect prospect that she did not consume at $t = 1$.

Note, a consumer is not forgetful of past *abstinence* in our model. That is, a consumer never mistakenly remembers consuming after choosing not to consume at $t = 1$. Such asymmetric forgetting is broadly consistent with experimental evidence that individuals are overwhelmingly more likely to forget an event that did occur than to falsely remember an event that did not occur.[5] In Ap-

[5] As one example, subjects in a study by McDermott (1996) were shown lists of words and later asked to recall

pendix A.2, we nonetheless analyze a variant of our model in which consumers are symmetrically forgetful of past consumption and past abstinence.

Returning to our model, the (true) expected value of a subscription to a consumer can now be expressed as

$$V(\phi) = \Big(\sum_{t=1}^{2} D_t \cdot \mathrm{E}[v_t|d_t = 1]\Big) - \phi \cdot D_{12} - k \cdot D_\tau. \tag{2}$$

Although it is expressed solely in terms of the penalty fee $\phi$ in (2), the expected value of a subscription will also sometimes be expressed as $V(\phi|\alpha)$, where $\alpha$ implicitly enters through $D_1$, $D_2$, $D_{12}$, and $D_\tau$.

Besides allowing imperfect memory through $\alpha$, we also allow consumers to imperfectly anticipate their forgetfulness where $\widetilde{\alpha} \in [0,\alpha]$ denotes a consumer's initial expectation, held prior to $t = 2$, regarding the true value of $\alpha$ when $t = 2$ arrives.[6] Thus, a consumer is *naive* regarding her forgetfulness if $\widetilde{\alpha} = 0 < \alpha$, *fully sophisticated* if $\widetilde{\alpha} = \alpha$, and *partially sophisticated* if $0 < \widetilde{\alpha} < \alpha$. Continuing to use a tilde to denote a consumer's initial *expectation* of a given value — as in, and based on $\widetilde{\alpha}$ — we also let $\widetilde{D}_t$, $\widetilde{D}_{12}$, and $\widetilde{D}_\tau$ denote a consumer's initial expectations of $D_t$, $D_{12}$, and $D_\tau$ (respectively).[7]

An objective of this paper is to analyze the market consequences of modern advances in consumption tracking technology. In the model, advances in consumption tracking technology may be understood as giving consumers (or a segment of consumers) access to — and thus, the option to use — consumption tracking, which may not have previously been available. As an alternate interpretation, advances in consumption tracking technology may be understood as reducing the tracking cost $k$ associated with checking one's past consumption (from a prohibitively high level) for these consumers. Before we study the effects of consumption tracking, however, we first analyze a benchmark version of the model without consumption tracking.

words from the lists. In turn, subjects forgot (did not recall) 10 to 25 presented words (in short- and long-term recall tasks, respectively) for every one non-presented word that was incorrectly recalled — even though the lists were specifically designed to induce false recall of a "lure" that was similar to the presented words. Also see Deese (1959) and Roediger and McDermott (1995) for related evidence.

[6] Our restriction to $\widetilde{\alpha} \leq \alpha$ is supported by experimental evidence that consumers generally underestimate their forgetfulness (Ericson, 2011).

[7] Since the $t = 1$ consumption decision is informed only by $\widetilde{\alpha}$, $\widetilde{D}_1 = D_1$ must hold, though $\widetilde{D}_2$, $\widetilde{D}_{12}$, and $\widetilde{D}_\tau$ may differ from $D_2$, $D_{12}$, and $D_\tau$ (respectively) when $\widetilde{\alpha} \neq \alpha$.

# 3 Benchmark Analysis without Consumption Tracking

To begin our benchmark analysis, we first characterize the consumption behavior of a consumer who has subscribed to the service but does not have access to consumption tracking, or as an alternate (yet equivalent) interpretation, a consumer for whom consumption tracking is accessible but prohibitively costly (Section 3.1). After that, we characterize the firm's optimal contract and resultant equilibrium behavior without consumption tracking (Section 3.2).

## 3.1 Consumption Profiles without Consumption Tracking

Assuming she has subscribed to the service, a consumer's consumption choices depend on her realized valuations of the service ($v_1$ and $v_2$), her actual and perceived forgetfulness ($\alpha$ and $\widetilde{\alpha}$), and the penalty fee ($\phi$). If there is no penalty fee ($\phi = 0$), the consumer will always consume in both consumption periods. With a positive penalty fee ($\phi > 0$), we can use backward induction to characterize consumption behavior, starting with the $t = 2$ decision. If the consumer did not consume at $t = 1$, then she will always consume at $t = 2$ because she is not at risk of accruing the penalty. If she consumed at $t = 1$, she may still consume at $t = 2$, but only if her valuation $v_2$ exceeds $(1-\alpha)\phi$, which is the penalty fee scaled by her belief of the likelihood of having consumed at $t = 1$. Naturally, the likelihood of consuming at $t = 2$ in this case is, all else equal, higher if the consumer is more forgetful (larger $\alpha$). The $t = 1$ consumption decision then depends on $v_1$ as well as the consumer's expectation — based on $\widetilde{\alpha}$ — of her future $t = 2$ consumption behavior conditional on her $t = 1$ choice. An exact mathematical characterization of these consumption decisions is provided in Appendix B.

Proposition 1 summarizes four possible consumption profiles in the benchmark model without consumption tracking. All proofs are in Appendix C.

**Proposition 1.** *For a consumer who subscribes to the service without consumption tracking:*

(i) [memory-based avoidance] *if* $\alpha < \frac{\phi-1}{\phi}$, *then* $D_1 = \frac{1}{2}$, $D_2 = \frac{1}{2}$, *and* $D_{12} = \widetilde{D}_{12} = 0$*;*

(ii) [abstinence-based avoidance] *if* $\widetilde{\alpha} > \frac{\sqrt{(\phi-1)^2+1}}{\phi}$, *then* $D_1 = 0$, $D_2 = 1$, *and* $D_{12} = \widetilde{D}_{12} = 0$*;*

(iii) [unintentional accrual] *if* $\widetilde{\alpha} \leq \frac{\phi-1}{\phi} < \alpha$, *then* $D_1 = \frac{1}{2}$, $D_2 = 1 - \frac{(1-\alpha)\phi}{2} > \frac{1}{2}$, *and* $D_{12} =$

$\frac{1-(1-\alpha)\phi}{2} > 0 = \widetilde{D}_{12}$*;*

(iv) [intentional accrual] *if* $\frac{\phi-1}{\phi} < \widetilde{\alpha} < \frac{\sqrt{(\phi-1)^2+1}}{\phi}$*, then* $D_1 = \frac{1+(\phi-1)^2-\widetilde{\alpha}^2\phi^2}{2}$*,* $D_2 = 1 - \frac{(1-\alpha)\phi(1+(\phi-1)^2-\widetilde{\alpha}^2\phi^2)}{2}$*,* $D_{12} = \frac{(1-(1-\alpha)\phi)(1+(\phi-1)^2-\widetilde{\alpha}^2\phi^2)}{2} > 0$*, and* $\widetilde{D}_{12} = \frac{(1-(1-\widetilde{\alpha})\phi)(1+(\phi-1)^2-\widetilde{\alpha}^2\phi^2)}{2} > 0$.

Parts (i) and (ii) of Proposition 1 describe two distinct scenarios in which a consumer always avoids incurring a penalty fee ($D_{12} = 0$). In part (i), the consumer may consume at $t = 1$ ($D_1 > 0$), after which she can — due to her low forgetfulness — rely on her memory of consuming at $t = 1$ to avoid consuming a second time at $t = 2$. Under this *memory-based avoidance* strategy, the probability of consumption is the same and equal to $\frac{1}{2}$ in each period.

Alternatively, part (ii) of Proposition 1 describes a strategy of *abstinence-based avoidance*. In this case, the consumer is too forgetful to rely on her memory as in part (i). However, she is sufficiently sophisticated (high enough $\widetilde{\alpha}$) to anticipate that she will be forgetful of any $t = 1$ consumption, making her susceptible to consuming in both periods by mistake. To avoid the penalty, she abstains from consumption at $t = 1$ while being free to consume at $t = 2$ without incurring a penalty.

Parts (iii) and (iv) of Proposition 1 describe scenarios in which consumers might accrue the penalty. Specifically, part (iii) describes a consumption profile characterized by *unintentional accrual* due to a high degree of forgetfulness coupled with low sophistication (i.e. high $\alpha$ and low $\widetilde{\alpha}$). In this case, the consumer initially *expects* to follow the memory-based avoidance strategy described in part (i), and likewise consumes at $t = 1$ with probability $\frac{1}{2}$. However, the consumer is too forgetful to follow through, and "accidentally" accrues the penalty with some positive probability ($D_{12} > 0 = \widetilde{D}_{12}$).

Part (iv) instead describes a consumption profile with *intentional accrual* in that the consumer may incur the penalty with $D_{12} > 0$, but now anticipates this possibility since $\widetilde{D}_{12} > 0$ (though $D_{12}$ and $\widetilde{D}_{12}$ may differ). The condition for intentional accrual, $\frac{\phi-1}{\phi} < \widetilde{\alpha} < \frac{\sqrt{(\phi-1)^2+1}}{\phi}$, suggests that it can apply to moderately sophisticated consumers. Furthermore, this condition is always satisfied if $\phi < 1$ (since $\phi < 1$ implies $\frac{\phi-1}{\phi} < 0$ and $1 < \frac{\sqrt{(\phi-1)^2+1}}{\phi}$), which means that the other three potential consumption profiles — memory-based avoidance, abstinence-based avoidance, and unintentional accrual — can *only* arise if $\phi \geq 1$.

Figure 1 illustrates the regions, depending on $\alpha$ and $\phi$, under which different consumption profiles may arise for naive consumers and for fully sophisticated consumers. The figure shows that a fully sophisticated consumer will not unintentionally accrue the consumption and pay the penalty fee mistakenly. Meantime, a naive consumer will not abstain from the consumption in the first period to avoid the penalty fee.

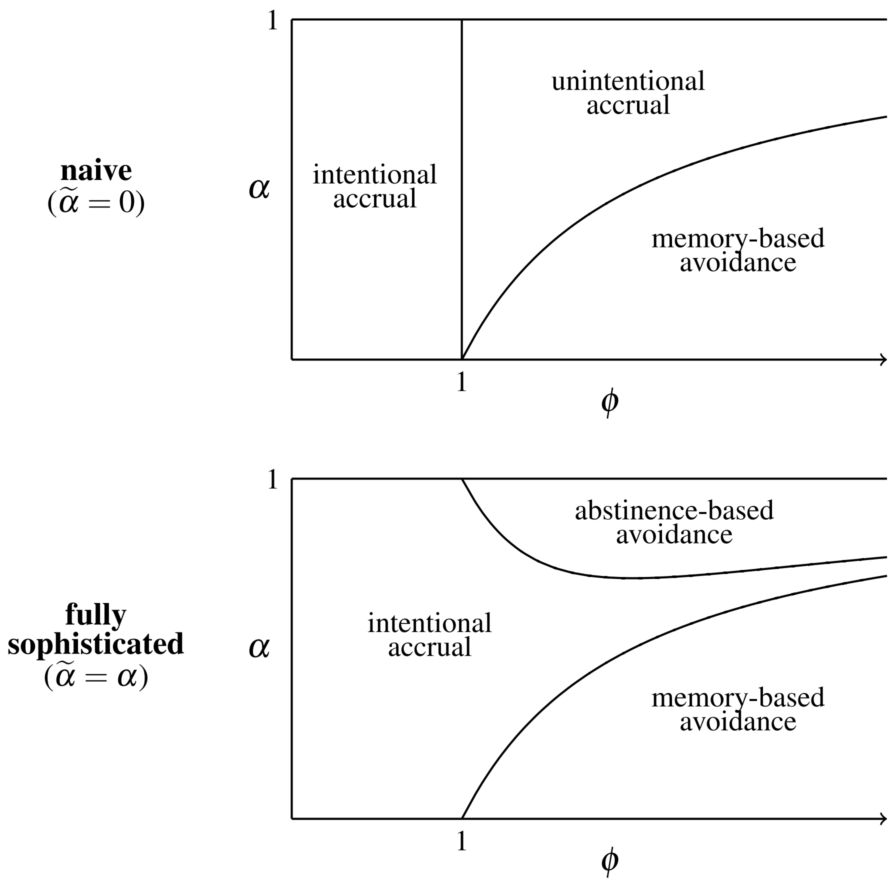


Figure 1: Consumption Profiles without Consumption Tracking

An interesting insight from Proposition 1 is that, for a consumer to (mistakenly) consume in both periods and incur a high penalty fee, being forgetful is not enough. A sufficiently sophisticated consumer can anticipate this cognitive limitation and abstain from consumption in the first period to avoid such mistakes (as in the case of abstinence-based avoidance). Thus, a consumer's level of sophistication (through $\widetilde{\alpha}$) is an important factor determining the dynamics of her consumption behavior. The following corollary summarizes the effect of $\widetilde{\alpha}$ on consumption in the benchmark model without consumption tracking:

**Corollary 1.** *Without consumption tracking, an increase in $\widetilde{\alpha}$ has the following effects (all else equal):*

(i) $D_1$ *decreases;*

(ii) $D_2$ *increases;*

(iii) $D_{12}$ *decreases.*

Corollary 1 implies that, in comparison to a less sophisticated consumer, a more sophisticated consumer (with higher $\widetilde{\alpha}$) tends to back-load her consumption by consuming less (on average) in the first period and more in the second. In addition, the more sophisticated consumer is less likely to accrue the penalty by consuming in both periods. Corollary 1 thus suggests a negative association between a consumer's tendency to accrue penalty fees and a tendency to back-load consumption. Consistent with this prediction, Liu et al. (2018) report that consumers who frequently overdraft their bank accounts also tend to *front*-load consumption in that they disproportionately spend from their accounts at the beginning of a pay period compared to the end of the pay period.

### 3.2 Benchmark Equilibria without Consumption Tracking

We now consider the firm's optimal contract in the benchmark model without consumption tracking. Using a subscript $NT$ (short for "no tracking") to connote benchmark equilibrium values, this contract is described by the optimal penalty fee, $\phi^*_{NT}$, and subscription price, $p^*_{NT}$, which is simply a consumer's maximum willingness-to-pay for a subscription given the penalty fee (i.e. $p^*_{NT} = V(\phi^*_{NT}|\widetilde{\alpha})$). Similarly, we will use $\Pi_{NT}$ and $CS_{NT}$ to denote, respectively, expected profits and expected consumer surplus in the benchmark equilibrium.

**Proposition 2.** *Without consumption tracking:*

(i) [zero-penalty benchmark] *If* $\alpha \leq \frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$*, then* $\phi^*_{NT} = 0$ *and* $p^*_{NT} = 1$*; in this equilibrium,* $D_{12} = \widetilde{D}_{12} = 1$*, while* $\Pi_{NT} = 1$ *and* $CS_{NT} = 0$*.*

(ii) [positive-penalty benchmark] *If* $\alpha > \frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$, *then* $\phi^*_{NT} = \max\left\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\right\}$ *and* $p^*_{NT} = \frac{5}{8}$*; in this equilibrium,* $D_{12} > \widetilde{D}_{12} = 0$*, while* $\Pi_{NT} = \frac{5}{8} + \frac{\mathrm{I}[\widetilde{\alpha} \leq 2\alpha - 1]}{8(1-\alpha)} + \frac{(\alpha - \widetilde{\alpha})\mathrm{I}[\widetilde{\alpha} > 2\alpha - 1]}{2(1-\widetilde{\alpha})^2} > 1$ *and* $CS_{NT} = \min\left\{\frac{1-3\alpha}{16(1-\alpha)}, \frac{\widetilde{\alpha}^2 - \alpha^2}{4(1-\widetilde{\alpha})^2}\right\} < 0$.

Part (i) of Proposition 2 describes a *zero-penalty benchmark equilibrium*, in which the firm sets its penalty fee to zero while extracting all consumer surplus through a relatively high subscription price ($p^* = \mathrm{E}[v_1 + v_2] = 1$) that reflects the full expected value from consuming in both periods without penalty. As seen, the zero-penalty benchmark equilibrium arises if consumers have sufficiently reliable memories (i.e. low enough $\alpha$). In this case, the firm would not profit from a high penalty fee because these consumers would engage in memory-based avoidance.

Interestingly, the zero-penalty benchmark equilibrium also arises with more forgetful consumers who are sufficiently sophisticated about their forgetfulness. With fully sophisticated consumers ($\widetilde{\alpha} = \alpha$), for instance, it is readily verifiable that the condition for the zero-penalty benchmark equilibrium must hold. These consumers would also successfully avoid a high penalty fee, except now through abstinence-based (instead of memory-based) avoidance.

Part (ii) of Proposition 2 describes a *positive-penalty benchmark equilibrium* that arises if consumers are sufficiently forgetful and not too sophisticated about their forgetfulness (i.e. $\alpha$ must be sufficiently high relative to $\widetilde{\alpha}$).[8] In this equilibrium, the firm charges a positive penalty fee while offering a lower subscription price than in the zero-penalty benchmark equilibrium. Here, the firm strategically sets its penalty fee at a level that leads consumers to *believe* they will succeed in memory-based penalty avoidance, but in reality consumers may mistakenly consume in both periods and thus unintentionally accrue the penalty.

By comparing the expressions for $\Pi_{NT}$ in parts (i) and (ii) of Proposition 2, we can see that the positive-penalty benchmark equilibrium entails higher profits than the zero-penalty benchmark equilibrium. This makes sense considering the positive-penalty benchmark equilibrium only exists when it is possible to improve upon the profits that would be attained without a penalty fee. Here, the potential to earn extra profits stems from the firm's ability to collect *unintentionally-accrued*

[8] As discussed earlier, there is ample empirical evidence that consumers can be quite forgetful and have only limited sophistication regarding their forgetfulness. See, in particular, footnotes 1-2 and surrounding discussions.

penalty fees. To the firm, it is important that the penalty fees are unintentionally accrued; otherwise, consumers would have to be compensated with an even lower subscription price that reflects the true value of a subscription. In this sense, a consumer overvalues, and thus *overpays* for a subscription in the positive-penalty benchmark equilibrium, leading to *negative* consumer surplus.

Next, recall that four consumption profiles could hypothetically arise with a positive penalty fee in Proposition 1. From Proposition 2, however, only the "unintentional accrual" profile can be supported in a market equilibrium with a positive penalty fee. This prediction — i.e. that unintentional accrual will be a feature of markets with positive penalty fees — fits with evidence that penalty fees are often unexpected. According to one survey, for instance, only 10% of consumers who overdraft their checking accounts do so intentionally (Pew Center on the States, 2012).[9]

The conditions on $\alpha$ in Proposition 2 also suggest that the use of (positive) penalty fees will be more common in contexts where consumers are more forgetful. For example, it is arguably harder — implying higher $\alpha$ — to (mentally) keep track of one's checking account balance than to keep track of one's usage of minutes on a calling plan. After all, tracking one's checking account likely requires remembering a wide range of spending "types" — hand-written checks, ATM withdrawals, debit card purchases (which may be easily confused with credit card purchases), preauthorized automatic payments (which may have minimal salience and thus be easily forgotten) — not to mention remembering the varying amounts of each expenditure. By contrast, a consumer's use of minutes on a calling plan is presumably more consistent, with each additional minute of calling likely to be similar in salience to the previous minute (and involving an easier "+1" calculation to track cumulative use). In light of this comparison, Proposition 2 would seem to indicate that penalty fees would be more common for checking accounts than for calling plans. Consistent with this prediction, overdraft fees have long been a staple of personal checking accounts yet "unlimited" calling plans (i.e. without penalty fees) are relatively common.

[9] A British Financial Conduct Authority (2008) study similarly indicates that only 7% of overdraft fees are intentionally accrued.

# 4 Analysis: The Effects of Consumption Tracking

In this section, we analyze the model *with* consumption tracking. In particular, we now suppose that a $\lambda \in (0,1]$ share of consumers have access to a consumption tracking technology that (as previously described) allows them to check, upon their arrival at $t = 2$, whether or not they consumed at $t = 1$.[10] We will refer to these consumers as *trackers* and the remaining $1-\lambda$ share of consumers as *non-trackers*. Note, that a tracker (according to our definition) simply has the *option* to use consumption tracking, but does not necessarily choose to use it. Meanwhile, a non-tracker does *not* get to make such a choice (or otherwise have the option) to use consumption tracking. Indeed, this could reflect a lack of access to consumption tracking (and we will sometimes lean on this interpretation). With that said, non-trackers do not necessarily lack access to the technology as they may equivalently be understood as consumers for whom the use of consumption tracking is prohibitively costly.

Indeed, the present characterization of consumer segments can be interpreted in various ways. As one possibility, trackers may be understood as early adopters of a consumption tracking technology, with non-trackers representing late- or non-adopters. Related to this, perhaps trackers are simply aware of the technology while non-trackers are not (maybe they were never aware or were once aware but have since forgot). Alternatively, consumption tracking may only be available to a subset of consumers, as could be the case if a tracking app was available on some mobile platforms but not others (e.g. Apple iOS but not Android, or vice versa). Lastly, and as alluded to above, the distinction between trackers and non-trackers may also be understood as reflecting heterogeneity in the *cost* of consumption tracking — with the understanding that all consumers have access to the technology — where the cost is presumed to be prohibitively high for non-trackers, but low enough for trackers to allow its potential use (under this interpretation, it will be implicit that $k$ represents the tracking cost for trackers).[11]

[10] This formulation allows us to consider settings ranging from those in which consumption tracking has relatively little market penetration (i.e. with small $\lambda > 0$) to the case of complete market penetration ($\lambda = 1$).

[11] In Appendix A.3, we analyze a more general version of the model in which the $1-\lambda$ share of consumers merely face a higher tracking cost — that may not be prohibitively costly — compared to the remaining $\lambda$ share of consumers.

### 4.1 The Tracking Decision

Our next lemma describes conditions under which trackers choose to track their $t = 1$ consumption.

**Lemma 1.** *A tracker chooses to track her consumption ($d_\tau = 1$) after consuming at $t = 1$ ($d_1 = 1$) if and only if $k < k_\tau(\alpha, \phi)$, where:*

$$k_\tau(\alpha,\phi) \equiv \begin{cases} \frac{\alpha(1-\alpha)\phi^2}{2}, & \phi \leq 1, \\ \frac{(1-\alpha)(\alpha\phi^2-(\phi-1)^2)}{2}, & 1 < \phi < \frac{1}{1-\alpha}, \\ \frac{\alpha}{2}, & \phi \geq \frac{1}{1-\alpha}. \end{cases} \tag{3}$$

Thus, consumption tracking may be used as long as the tracking cost $k$ does not exceed a threshold $k_\tau$ given in (3). It is readily verifiable that $k_\tau$ (weakly) increases with $\phi$, which means that a consumer is willing to incur a greater cost to track her consumption when the penalty fee is larger. The relationship between $k_\tau$ and $\alpha$, however, is non-monotonic. For instance, a consumer with perfect memory ($\alpha = 0$) and a consumer with no memory ($\alpha = 1$) of past consumption are both unwilling to incur *any* cost to track their consumption (i.e. $k_\tau = 0$ for $\alpha = 0, 1$), as both believe (one correctly, the other incorrectly) that they already know whether or not they consumed at $t = 1$. Consumers with $0 < \alpha < 1$, however, are willing to incur some cost to track their consumption in the presence of a positive penalty fee (i.e. $k_\tau > 0$ if $0 < \alpha < 1$ and $\phi > 0$).

With $\widetilde{\alpha} < \alpha$, the true threshold tracking cost $k_\tau(\alpha, \phi)$ may differ from a tracker's ex-ante *perception* of this threshold, $k_\tau(\widetilde{\alpha}, \phi)$. This implies two possible scenarios in which a tracker may mispredict her future use of consumption tracking. First, if $k_\tau(\widetilde{\alpha}, \phi) \leq k < k_\tau(\alpha, \phi)$, a tracker will choose to track her $t = 1$ consumption despite a prior belief that she would not use consumption tracking ($D_\tau > \widetilde{D}_\tau = 1$). Conversely, if $k_\tau(\alpha, \phi) \leq k < k_\tau(\widetilde{\alpha}, \phi)$, a tracker expects to track her $t = 1$ consumption, but decides against it when $t = 2$ arrives ($D_\tau = 0 < \widetilde{D}_\tau$). We refer to these two scenarios as unplanned use and unplanned nonuse, respectively.

Figure 2 illustrates how a tracker's expected and actual use of consumption tracking may vary depending on her forgetfulness and her level of sophistication.[12] As seen in the light green

[12] This illustration uses $\phi = 1$ and $k = .1$. While their precise positions may differ, the same general regions would

region (in which $k < k_\tau(\alpha, \phi)$), here only moderately forgetful consumers actually track their $t = 1$ consumption. However, these consumers might not plan to use consumption tracking if they have low sophistication, while the most forgetful consumers with moderate sophistication may falsely expect to use consumption tracking, as seen in the portion of the dotted blue region (where $k < k_\tau(\widetilde{\alpha}, \phi)$) that does not overlap with the light green region.

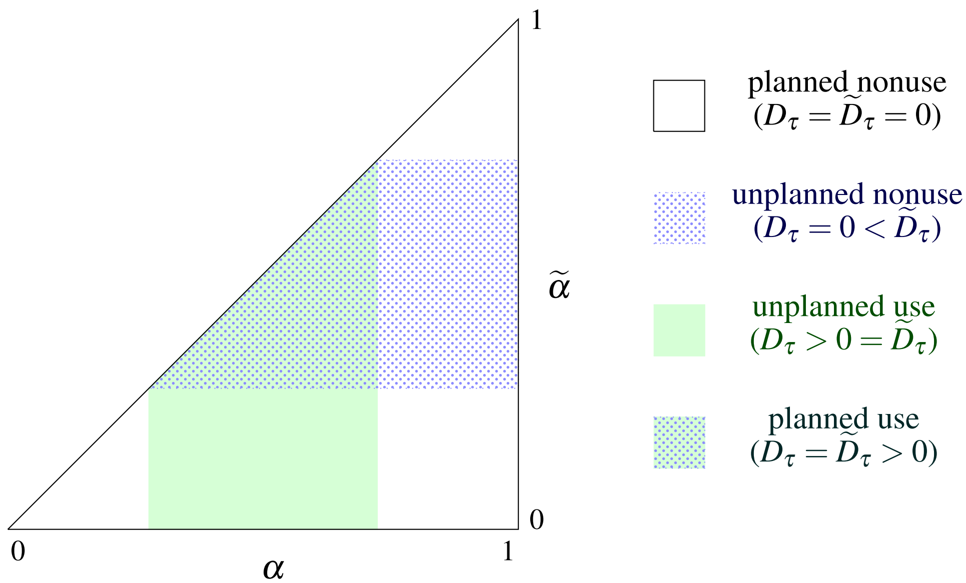


Figure 2: A Tracker's Actual and Perceived Use of Consumption Tracking

In our remaining analysis, it will sometimes be useful to express the tracking condition in terms of $\phi$ instead of $k$. For this purpose, we define

$$\phi_\tau(\alpha, k) \equiv \{\phi : k = k_\tau(\alpha, \phi)\}, \tag{4}$$

which is the threshold penalty fee at which a tracker who consumed at $t = 1$ would be indifferent between tracking and not tracking her consumption. Implicitly, $\phi_\tau$ is only defined for $k \leq \frac{\alpha}{2}$ because, as we saw in (3), $k > \frac{\alpha}{2}$ implies $k > k_\tau$ for all $\phi$. Recalling that $k_\tau$ is increasing in $\phi$, it follows that the condition for consumption tracking ($k < k_\tau$) can equivalently be expressed as $\phi > \phi_\tau$ (where $\phi_\tau$ is defined).

exist for other values of $\phi$ and $k$, unless $k \geq \frac{1}{8}\left(1 + \frac{\phi - 1}{\max\{\phi, 1\}}\right)^2$, in which case a tracker would (for any $\alpha$ and $\widetilde{\alpha}$) never track or expect to track their $t = 1$ consumption.

## 4.2 Equilibria with Consumption Tracking

We start with the case in which the availability of consumption tracking has no effect on equilibrium behavior.

**Proposition 3.** *If $k$ is sufficiently large or $\alpha$ is sufficiently small (or both), then the equilibrium with consumption tracking is identical to the benchmark equilibrium without consumption tracking.*

Thus, the benchmark equilibrium will be maintained when the consumption tracking technology is not valuable to consumers — either if $k$ is sufficiently large or if $\alpha$ is sufficiently small. With a large $k$, the availability of consumption tracking has no effect on equilibrium behavior because consumption tracking is too costly for trackers to use (or expect to use) the technology. With small $\alpha$, consumers have reliable memories and would be adept at avoiding penalty fees even without consumption tracking; as a result, the firm optimally sets its penalty fee to zero with or without consumption tracking.

**Proposition 4.** *If $\lambda$ is sufficiently small and $\phi_{NT}^* > 0$, then $\phi^* = \phi_{NT}^*$ and $p^* = p_{NT}^*$. However, the equilibrium with consumption tracking may still differ from the associated benchmark equilibrium as $D_\tau > 0$ if $k_\tau(\widetilde{\alpha}, \phi_{NT}^*) \leq k < k_\tau(\alpha, \phi_{NT}^*)$, while only non-trackers subscribe to the service if $k < k_\tau(\widetilde{\alpha}, \phi_{NT}^*)$. In both cases, $\Pi < \Pi_{NT}$ and $CS > CS_{NT}$.*

Proposition 4 considers a situation in which the positive-penalty benchmark equilibrium would arise without consumption tracking, and where the share of trackers is relatively small. Under these conditions, the firm does not adjust its penalty fee or subscription price in response to the availability of consumption tracking. In the equilibrium, the trackers benefit from the availability of consumption tracking technology, but the positive effect does not extend to the non-trackers. As one possibility, with a fairly low tracking cost, trackers may now make use of the technology to avoid penalty fees. In this case, the availability of consumption tracking *directly* helps these consumers at the expense of the firm, as it leads to higher consumer surplus and lower profits.

With an even lower tracking cost, trackers will no longer be served in equilibrium. In this case, trackers (incorrectly) expect that they would succeed in memory-based avoidance of the penalty fee, even if they did *not* use consumption tracking. Nonetheless, trackers would still expect to track their consumption — and thus incur the tracking cost $k$ — with some positive probability, which reduces their willingness-to-pay for a subscription. Since the share of trackers is small, however, the firm chooses to maintain its benchmark subscription price (while only serving non-trackers) instead of lowering its price to accommodate trackers. With fewer subscribers, the firm's profits are lower than in the positive-penalty benchmark equilibrium even though the optimal contract does not change. Meanwhile, consumer surplus is higher because unserved trackers now avoid the *negative* expected surplus from subscribing to the service.

**Proposition 5.** *If $\lambda$ is sufficiently large and $k$ is sufficiently small with $\phi^*_{NT} > 0$, then $\phi^* = \phi_\tau(\alpha,k) \cdot \mathrm{I}\big[\alpha > 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi^2_\tau(\alpha,k)-4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k))+4(1-\widetilde{\alpha}^2)\phi^2_\tau(\alpha,k)}\big] < \phi^*_{NT}$ and $p^* = V(\phi^*|\widetilde{\alpha}) > p^*_{NT}$. In this equilibrium, $\Pi < \Pi_{NT}$ and $CS > CS_{NT}$, while $D_\tau = \widetilde{D}_\tau = 0$*

Like Proposition 4, Proposition 5 considers a situation in which the positive-penalty fee would arise without consumption tracking, except now the share of trackers is presumed to be large while the tracking cost is small. In this case, the availability of consumption tracking compels the firm to *reduce* (and possibly eliminate) its penalty fee. Specifically, the firm reduces its penalty fee to a point where trackers are no longer inclined to use the technology, as trackers — who are now quite prevalent — would have used consumption tracking to avoid penalty fees (thus undermining the firm's profits) if the firm maintained its penalty fee from the positive-penalty benchmark equilibrium. If consumers are sufficiently forgetful (high enough $\alpha$), the firm still uses a positive penalty fee, but with less forgetful consumers it becomes more profitable for the firm to eliminate its penalty fee altogether.

The effect of tracking technology in Proposition 5 is likely the ideal case for regulators – a technology that substantially reduces the tracking cost and is accessible to a sufficiently large segment of market. By forcing the firm to reduce or eliminate its penalty fee, the availability of consumption tracking in Proposition 5 leads to lower profits and higher consumer surplus even though consumers do not actually use the technology. In this way, the result shows how consumers — including non-trackers without access to consumption tracking — can *indirectly* benefit from the availability of consumption tracking when the technology is widely accessible and the cost of using it is sufficiently low.

So far, our analysis of equilibria with consumption tracking has focused on the roles of the tracking cost ($k$), the share of trackers ($\lambda$), and consumers' level of forgetfulness ($\alpha$). Our next two corollaries instead address the role of consumer sophistication ($\widetilde{\alpha}$).

**Corollary 2.** *If consumers are fully sophisticated ($\widetilde{\alpha} = \alpha$), the zero-penalty benchmark equilibrium arises for all $\lambda \in [0,1]$*

Thus, with fully sophisticated consumers, the firm never uses a positive penalty fee, regardless of the availability of consumption tracking.

**Corollary 3.** *If consumers are naive ($\widetilde{\alpha} = 0$), then $\phi^* \leq \phi^*_{NT}$, $p^* \geq p^*_{NT}$, $\Pi \leq \Pi_{NT}$, $CS \geq CS_{NT}$, and $D_\tau \geq 0 = \widetilde{D}_\tau$.*

With naive consumers, the availability of consumption tracking can thus only result in: a lower penalty fee; a higher subscription price; lower profits; and higher consumer surplus. Indeed, these effects are consistent with the effects of consumption tracking seen in previous results. Furthermore, naive consumers never expect to use consumption tracking, even though they may change their mind when $t = 2$ arrives.

At this point, we have yet to address the effects of consumption tracking with *moderate* tracking costs and *partially* sophisticated consumers. As the next result reveals, these conditions can give rise to a fundamentally different type of equilibrium in which trackers *expect* to use consumption tracking but do not actually use it.

**Proposition 6.** *There exist a $k_L$, $k_H$, $\widetilde{\alpha}_L$, and $\widetilde{\alpha}_H$ with $k_L \leq k_H$ and $\widetilde{\alpha}_L \leq \widetilde{\alpha}_H$ (non-binding for some $\alpha$, $\lambda$) such that, if $k_L < k < k_H$ and $\widetilde{\alpha}_L < \widetilde{\alpha} < \widetilde{\alpha}_H$ with sufficiently large $\lambda$, then $\phi^* = \phi_\tau(\alpha, k)$ and $p^* = \frac{5-4k(1-k)}{8}$, while $D_\tau = 0 < \widetilde{D}_\tau$.*

Proposition 6 thus shows that an *unplanned nonuse equilibrium* — referring to an equilibrium in which consumers expect to use consumption tracking but do not actually use it (i.e. with $D_\tau = 0 < \widetilde{D}_\tau$) — can arise with moderate tracking costs and moderately sophisticated consumers.

Reviewing our prior results, we can see that an unplanned nonuse equilibrium can in fact *only* arise under these circumstances.

One unique — and perhaps surprising — potential outcome in an unplanned nonuse equilibrium is that the firm's profits may be *higher* than in the corresponding benchmark equilibria without consumption tracking. This possibility — i.e. that consumption tracking may actually *help* the firm — is evident in our next two closely-related examples; derivations of the optimal contracts (and other equilibrium measures) in these examples are provided in Appendix D.

**Example 1.** *Let* $\alpha = \frac{9}{10}$, $\widetilde{\alpha} = \frac{25}{28}$, $k = \frac{13}{100}$, *and* $\lambda = 1$. *Then* $\phi^* = \phi_\tau(\alpha, k) = 2 > \phi^*_{NT} = 0$ *and* $p^* = \frac{11{,}369}{20{,}000} < p^*_{NT} = 1$. *In this equilibrium,* $\Pi = \frac{23{,}209}{20{,}000} > \Pi_{NT} = 1$, $CS = -\frac{3{,}663}{10{,}000} < C_{NT} = 0$, *and* $D_\tau = 0 < \widetilde{D}_\tau = \frac{37}{100}$.

Example 1 reveals an unplanned nonuse equilibrium with several noteworthy properties. To begin, observe that this equilibrium features a positive penalty fee even though the firm would have set its penalty fee to zero in the absence of consumption tracking ($\phi^* > 0 = \phi^*_{NT}$). Thus, in this case, the availability of consumption tracking compels the firm to impose a positive penalty fee that would *not otherwise exist*.

The reason that a positive penalty is optimal with — but not without — consumption tracking in Example 1 stems from the fact that the availability of consumption tracking can instill a false belief in a tracker that she will use the technology to avoid penalty fees. That is, a tracker may choose to consume at $t = 1$ — followed by the possibility of mistaken consumption and accrual of the penalty fee at $t = 2$ — based on this incorrect belief; if consumption tracking had not been available, however, the consumer would not have been confident in her ability to avoid penalty fees after consuming in the first period, and would therefore abstain from consumption at $t = 1$ — in effect, pursuing abstinence-based avoidance of the penalty fee (i.e. only consuming at $t = 2$). In other words, by giving consumers a false sense of security — manifest as an *incorrect* belief that they might use consumption tracking to avoid the penalty — the availability of consumption tracking can, rather ironically, *prevent* consumers from avoiding penalty fees in an unplanned nonuse equilibrium.

As alluded to earlier, the equilibrium in Example 1 involves higher profits in comparison to the (zero-penalty) benchmark equilibrium that would arise in the absence of consumption tracking. The equilibrium also involves lower consumer surplus. Thus, unlike the other types of equilibria (i.e. besides unplanned nonuse) examined in our prior analysis, here the availability of consumption tracking *helps* the firm and *hurts* consumers. While these effects may seem surprising from the standpoint that consumption tracking is supposed to help consumers avoid penalty fees, they are natural consequences of the fact that the availability of consumption tracking compels the firm to impose a positive penalty fee that would not otherwise exist.

**Example 2.** *Let* $\widetilde{\alpha} = \frac{8}{9}$*, with* $\alpha$*, k, and* $\lambda$ *as given in Example 1. Then* $\phi^* = \phi_\tau(\alpha, k) = 2 < \phi^*_{NT} = 9$ *and* $p^* = \frac{11{,}369}{20{,}000} < p^*_{NT} = \frac{5}{8}$*. In this equilibrium,* $\Pi = \frac{23{,}209}{20{,}000} > \Pi_{NT} = \frac{43}{40}$*,* $CS = -\frac{3{,}663}{10{,}000} > C_{NT} = -\frac{161}{400}$*, and* $D_\tau = 0 < \widetilde{D}_\tau = \frac{37}{100}$*.*

Example 2 is, in many ways, similar to Example 1. While $\widetilde{\alpha}$ is slightly smaller than before, all other parameter values remain the same. Moreover, the availability of consumption tracking in Example 2 gives rise to the same unplanned nonuse equilibrium from Example 1 (note, $\phi^*$, $p^*$, $\Pi$, $CS$, and $D_\tau$ are the same in both examples). The key difference, however, is that a different benchmark equilibrium would have arisen in the absence of consumption tracking. More precisely, the reduction in $\widetilde{\alpha}$ from Example 1 to Example 2 leads to a reduction in the threshold value of $\alpha$ from Proposition 2, which defines the boundary between the zero-penalty and positive-penalty benchmark equilibria. Thus, while $\alpha$ was below this threshold in Example 1 — implying a zero-penalty benchmark equilibrium without consumption tracking — $\alpha$ is above this threshold in Example 2. As a result, a *positive*-penalty benchmark equilibrium would now arise without consumption tracking.

Even though Examples 1 and 2 feature the same unplanned nonuse equilibrium, they may reflect qualitatively different effects of consumption tracking since the corresponding benchmark equilibria are different. Notably, and in contrast to Example 1, the optimal penalty fee in Example 2 is now lower with consumption tracking than without. While we are now back to a "normal" situation in which the availability of consumption tracking compels the firm to reduce its penalty fee, the unplanned nonuse equilibrium in Example 2 is still anomalous in other regards. First, unlike all other equilibria considered thus far, the penalty fee and the subscription price move in the *same* direction, with the firm reducing both its subscription price and its penalty fee in response to the availability of consumption tracking. Here, it may seem counter-intuitive that the firm would need to lower its subscription price even with a lower penalty fee. However, the availability of consumption tracking reduces consumers' willingness-to-pay for a subscription as they now expect to incur the cost $k$ to track their consumption.

Even though the firm in Example 2 lowers its penalty fee *and* its subscription price in response to the availability of consumption tracking, its profits still increase (as was the case in Example 1). Here, the firm can attain higher profits because consumers — guided by their false belief that they will use consumption tracking to avoid penalty fees — are more likely to consume in both periods (compared to the positive-penalty benchmark equilibrium), resulting in higher total payments of the penalty fee.

Notably, the increase in profits does *not* come at the expense of consumers in Example 2. Instead, the availability of consumption tracking is *mutually beneficial*. In this case, both profits and consumer surplus may increase because the contract generates more total value (from consumers' realized valuations of the service in periods with consumption) due to consumers' increased total use of the service. While consumers do end up paying more to the firm, this loss is offset by the added value from their increased use of the service.

**Corollary 4.** *Each of the following relations can only hold in an unplanned nonuse equilibrium with* $D_\tau = 0 < \widetilde{D}_\tau$*:*

(i) $\phi^* > \phi^*_{NT}$*;*

(ii) $\Pi > \Pi_{NT}$*;*

(iii) $CS < CS_{NT}$.

Our previous examples illustrated how the availability of consumption tracking can, perhaps surprisingly, lead to: a higher penalty fee (Example 1); higher profits (Examples 1 and 2); and/or lower consumer surplus (Example 1). In turn, Corollary 4 asserts that each of these effects can *only* be realized in cases where the availability of consumption tracking gives rise to an unplanned nonuse equilibrium in which trackers never use consumption tracking despite an initial belief that they might. Furthermore, since profits can only decrease in other types of equilibria, it also follows that the availability of consumption tracking can only be mutually beneficial to the firm and consumers (as in Example 2) if it leads to an unplanned nonuse equilibrium.

It is worth reiterating that an unplanned nonuse equilibrium — and by extension, the effects described in Corollary 4 — can only arise with *partially* sophisticated consumers.[13] If consumers were fully sophisticated, they would accurately forecast their future use of consumption tracking, thus ruling out any form of "unplanned" equilibrium. Meanwhile, if consumers were naive, they would never expect to use consumption tracking — and thus never experience the false sense of security that consumption tracking may allow them to avoid penalty fees.

A limitation of the present analysis is that it is largely framed in terms of the *possibility* of an unplanned nonuse equilibrium. It is therefore fair to wonder whether an unplanned nonuse equilibrium is a likely market outcome or just a mere theoretical possibility. Indeed, numerical integrations across our parameter space indicate that, in cases where the availability of consumption tracking leads the firm to change its penalty fee, an unplanned nonuse equilibrium will arise 8.4% of the time and 13.8% of the time conditional on the new penalty fee being positive.[14] Naturally, an unplanned nonuse equilibrium is only viable if a significant share of consumers have access to consumption tracking. In light of this, it is perhaps not surprising that the incidence of unplanned nonuse equilibria is higher if we further restrict our calculation to cases in which consumption tracking is available to all consumers (i.e. with $\lambda = 1$), as the incidence of unplanned equilibria rises to 16.2% over all such cases and to 29.9% if we condition on the new penalty fee being positive. In sum, while an unplanned nonuse equilibrium is not the most likely outcome according to

[13] This feature lends credence to the potential empirical viability of the unplanned nonuse equilibrium. As we recall, partial sophistication is an empirically-supported aspect of consumer decision-making (see footnote 2 and surrounding discussion).

[14] In this exercise, the reason we only consider cases in which the availability of consumption tracking compels the firm to change its penalty fee stems from the fact that $k$ is unbounded. Namely, the exercise becomes trivial when integrating across all possible $k$ because consumption tracking is irrelevant when $k$ is sufficiently large. Restricting our calculation to cases in which the firm changes its penalty fee thus allows us to omit the limitless number of cases in which consumption tracking is irrelevant as a result of $k$ being too large.

the measures considered here, we can see that it is certainly more than a remote possibility.

To relate our (entire) analysis of consumption tracking to observed practice, we may consider the evolution of personal banking in the United States. Seeing that overdraft fees have long been a staple of personal checking accounts, this market would naturally correspond to the positive-penalty benchmark equilibrium — along the lines described in part (ii) of Proposition 2 — prior to the development of modern tracking technologies.

The advent of online banking near the turn of the 21st century represented a major innovation for tracking consumer spending.[15] Initially, however, online banking was not too popular, with adoption by only 18% of internet-using consumers in 2000; similarly, mobile banking apps (another significant innovation) had a slow start, with the same 18% rate of adoption by 2011 (Fox, 2013). Meanwhile, the now popular *Mint* personal finance app had only attained 1 million registered users as of 2009 compared to 29 million in 2021 (Beersheba Research, 2021).

Indeed, from around 2000 until the mid 2010s, modern tracking technologies for personal banking were available but had yet to gain significant traction. In light of this, the industry during this period would naturally be characterized by low $\lambda$. According to Proposition 4, a firm would not adjust its (positive) penalty fee in response to the availability of consumption tracking if $\lambda$ is sufficiently low. Consistent with this prediction, banks' overdraft fees remained relatively flat during this time.[16]

Despite the relatively slow start, tracking technologies in personal banking have since become quite popular. By 2022, 89% of all consumers are using mobile banking apps (Yuen, 2022), while the Mint app alone (as mentioned above) had amassed 29 million registered users by 2021. These trends suggest that $\lambda$ has become quite large since the mid 2010s. During this time, there has also been a considerable shift from older web-based banking services to mobile apps that are optimized for easy use and convenient access (Centric Digital, 2016). This latter trend suggests an overall reduction in the tracking cost $k$. According to our analysis, a firm would reduce or eliminate

[15] Internet banking applications originated in the US in 1996 and were offered by large banks (Citibank, Wells Fargo) beginning in 2002 (Şanlı and Hobikoğlu, 2015).

[16] In 2015, for instance, average overdraft fees (after adjusting for inflation) were on par with their levels prior to the advent of modern tracking technologies — slightly higher than their 1999 levels and slightly lower than their 2000 levels (Statista Research Department, 2022).

its penalty fee relative to its benchmark level when $\lambda$ is sufficiently large and also as $k$ decreases (Propositions 5 and 6).[17] Consistent with these predictions, some banks have recently bucked longstanding practice by offering checking accounts without overdraft fees, while overdraft fees have declined considerably on accounts where these fees still exist.[18]

Note that the above discussion draws on our result in Proposition 6, which characterizes the possibility of an unplanned nonuse equilibrium. Empirical observation suggests that such "unplanned nonuse" may indeed be a characteristic of personal banking now that tracking technologies have become widespread. For instance, even though 89 percent of consumers are using mobile banking apps in 2022 (as noted above), consumers seem to make little use of the tracking services offered by these apps. According to an Ipsos-Forbes study (Strohm, 2021), only 5 percent of consumers identify "budgeting and tracking tools" as having been the most valuable feature of their mobile banking apps based on their use in the past year; this put "budgeting and tracking tools" as only the eighth most valued feature of such apps — well behind other features like "mobile check deposit" and "bill pay." Similarly, of the 29 million registered Mint users (as of 2021), only 3.6 million actively use the app on a monthly basis (Beersheba Research, 2021). Crucially, the fact that the inactive users once registered for the app lends credence to the idea that they intended to use the app but failed to follow through — consistent with our notion of unplanned nonuse.

## 5 Conclusion

This paper studies the market consequences of advances in consumption tracking technologies, including their effects on a firm's marketing strategies and on the welfare of consumers. Importantly, consumers in our model may be forgetful about their past consumption and have limited sophistication about their future forgetfulness. Consumers' forgetfulness and limited sophistication can

[17] While it is evident from the formal statements of Propositions 5 and 6 that both require sufficiently large $\lambda$ and is also explicit that Proposition 5 requires sufficiently small $k$, the prediction of a lower penalty fee resulting from a decrease in $k$ is implicit in Proposition 6 as well. This follows because $\phi^* = \phi_\tau(\alpha, k)$ and $k$ move in the same direction.

[18] In particular, average overdraft fees (after adjusting for inflation) decreased by 7.1 percent from 2015 to 2021 (or by $2.57 in 2021 dollars); see Statista Research Department (2022). Also see Bennett (2002) for background on the recent decisions of some banks (such as Capital One and Citibank) to eliminate overdraft fees. Interestingly, some telecoms have likewise introduced mobile data plans without overage fees in recent years (Newswire, 2019).

make them especially susceptible to penalty fees, while consumption tracking technologies offer a potential means to mitigate the problem. Our analysis demonstrates that the availability of consumption tracking may help consumers directly as they can use the technology to avoid penalty fees. The availability of consumption tracking technologies may also indirectly help consumers — even those without access to the technology — by forcing the firm to reduce or even eliminate its penalty fees.

Our analysis also reveals that the availability of consumption tracking may instill a false sense of security among consumers who are partially sophisticated about their forgetfulness. In particular, the firm may strategically set a penalty fee such that consumers expect to use consumption tracking to avoid penalty fees, but ultimately do not to bother. Notably, these consumers would have been more careful to avoid penalty fees (by abstaining from first-period consumption) if consumption tracking had not been available. In this way, the illusion of security created by consumers' access to the technology can, ironically, make consumers more vulnerable to penalty fees while allowing the firm to increase its profits. The intuition for this effect is consistent with consumer overconfidence about future self-control generated by a cancellation option in a contract (DellaVigna, 2009) as well as more general notions of excessive optimism with new technology (Clark et al., 2016).

Our results have direct implications for managers and regulators. Our results show that the firm's optimal targeting (e.g., whether to serve only trackers, only non-trackers, or both segments) and pricing (subscription price and penalty fee) depend on the diffusion of tracking technology and the degree of consumer forgetfulness. Managers may need to measure the forgetfulness and sophistication of their consumers and set their penalty fees accordingly. Managers should also be attuned to any changes in consumers' tracking costs and anticipate how the penalty fee, in addition to being a tool to exploit consumer forgetfulness, may affect consumers' use of consumption tracking technologies. Despite recent advances in consumption tracking technologies, these tracking costs (whether monetary or non-monetary) remain significant for many consumers. For example, using a mobile app to monitor one's account balances can entail access costs (e.g. the cost of a smartphone), data storage costs (e.g. for a smartphone with limited memory), learning costs (e.g. to learn how to use the app), mobile data usage costs, as well as costs related to the time and attention required to use the technology. The use of these technologies may also involve data security concerns; after all, giving a third-party app access to personal transaction information can have serious consequences. Even with a relatively low tracking cost, our analysis suggests that a consumer may not use a consumption tracking app due to the firm strategically setting its penalty fee at a level for which consumers lack a sufficient incentive to use the technology. In total, we can see that many hurdles must be overcome for a consumer to make regular use of the app.

For regulators, encouraging new consumption tracking technologies makes intuitive sense as a way of helping forgetful consumers track their consumption to reduce unnecessary penalty fees. However, the effects of such measures may not be as straightforward after taking into account consumers' tracking costs and the firm's strategic responses. Regulators should therefore carefully assess these factors. Overall, the potential benefits to consumers from advances in consumption tracking technologies will not be significant until enough consumers have adopted these technologies and the tracking costs have decreased substantially.

To further understand the effects of consumption tracking technologies in markets with forgetful consumers, future research may examine additional consumer decisions or the role of competition. For instance, banks and wireless service providers often offer service options with different levels of penalty fees. Examining the equilibrium design of service options may require additional assumptions of consumer heterogeneity in forgetfulness and sophistication. Consumer forgetfulness may also affect consumer decisions in choosing between service providers, and the availability of consumption tracking technologies may affect the intensity of price competition between service providers. On the empirical side, future research may develop hypotheses from our propositions and test them with observed behavior. For instance, empirical work may investigate the variations in the penalty fees during the diffusion process of a new tracking technology, or after the introduction of new regulations that affect the use or availability of consumption tracking technologies. At the consumer level, empirical research may also help assess whether (and if so, the extent to which) access to consumption tracking technologies creates an "illusion of security" among consumers.

## References


Dan Acland and Matthew R Levy. Naiveté, projection bias, and habit formation in gym attendance. *Management Science*, 61(1):146–160, 2015.

Steffen Altmann, Christian Traxler, and Philipp Weinschenk. Deadlines and memory limitations. *Management Science, forthcoming*, 2021.

Eva Ascarza, Anja Lambrecht, and Naufel Vilcassim. When talk is "free": The effect of tariff structure on usage under two- and three-part tariffs. *Journal of Marketing Research*, 49:882–899, 2012.

Ned Augenblick and Matthew Rabin. An experiment on time preference and misprediction in unpleasant tasks. *Review of Economic Studies*, 86(3):941–975, 2019.

Adib Bagh and Hemant K Bhargava. How to price discriminate when tariff size matters. *Marketing Science*, 32(1):111–126, 2013.

Beersheba Research. Intuit: Monetizing its vast data treasure trove by creating better outcomes for customers. *Seeking Alpha*, 2021. URL `https://seekingalpha.com/article/4470721-intuit-intu-stock-monetizing-vast-data-better-outcomes-for-customers`.

Karen Bennett. Banks that have cut or eliminated overdraft fees. *Bankrate*, 2002. URL `https://www.bankrate.com/banking/checking/banks-eliminated-overdraft-fees/`.

Centric Digital. Mobile banking strategy: 4 best practices to follow. *Centric Digital*, 2016. URL `https://www.centricdigital.com/blog/mobile-banking-strategy-best-practices`.

CFPB. Data point: Overdraft/nsf fee reliance since 2015 — evidence from bank call reports. Technical report, Consumer Financial Protection Bureau, 2021.

Jiawei Chen, Lai Jiang, and Saad Andalib Syed Shah. An empirical model of the effects of "bill shock" regulation in mobile telecommunication markets. *International Journal of Industrial Organization*, pages 1–24, 2022.

Yuxin Chen, Ganesh Iyer, and Amit Pazgal. Limited memory, categorization, and competition. *Marketing Science*, 29(4):650–670, 2010.

Brent B Clark, Christopher Robert, and Stephen A Hampton. The technology effect: how perceptions of technology drive excessive optimism. *Journal of Business and Psychology*, 31(1): 87–102, 2016.

James Deese. On the prediction of occurrence of particular verbal intrusions in immediate recall. *Journal of experimental psychology*, 58(1):17, 1959.

Stefano DellaVigna. Psychology and economics: Evidence from the field. *Journal of Economic Literature*, 47:315–372, 2009.

Stefano DellaVigna and Ulrike Malmendier. Paying not to go to the gym. *american economic Review*, 96(3):694–719, 2006.

Preyas S Desai, Devavrat Purohit, and Bo Zhou. Allowing consumers to bundle themselves: The profitability of family plans. *Marketing Science*, 37(6):953–969, 2018.

Peter R. Dickson and Alan G. Sawyer. The price knowledge and search of supermarket shoppers. *Journal of Marketing*, 54(3):42–53, 1990.

Keith M. Marzilli Ericson. Forgetting we forget: Overconfidence and memory. *Journal of the European Economic Association*, 9(1):43–60, 2011. ISSN 15424766, 15424774.

FCA. How overdrafts are managed. Technical report, Financial Conduct Authority, 2008.

Gadi Fibich, Roy Klein, Oded Koenigsberg, and Eitan Muller. Optimal three-part tariff plans. *Operations Research*, 65(5):1177–1189, 2017.

Susannah Fox. 51% of u.s. adults bank online. Report 1, The Pew Center on The States, 2013.

Michael D. Grubb. Consumer inattention and bill-shock regulation. *Review of Economic Studies*, 82:219–257, 2015.

Michael D. Grubb and Matthew Osborne. Cellular service demand: Biased beliefs, learning, and bill shock. *American Economic Review*, pages 234–271, 2015.

Liang Guo. Overage charge or loyalty discount: When should consumptions be penalized or rewarded? *Marketing Science, Forthcoming*, 2022.

James G Helgeson and Sharon E Beatty. Price expectation and price recall error: An empirical study. *Journal of Consumer Research*, 14(3):379–86, 1987.

Fabian Herweg and Konrad Mierendorff. Uncertain demand, consumer loss aversion, and flat-rate tariffs. *Journal of the European Economic Association*, 11(2):399–432, 2013.

Anja Lambrecht, Katja Seim, and Bernd Skiera. Does uncertainty matter? consumer behavior under three-part tariffs. *Marketing Science*, 26:698–710, 2007.

Peter Landry. Sunk 'decision points': a theory of the endowment effect and present bias. *Theory and Decision*, 86(1):23–39, 2019.

Peter Landry. Pricing, advertising, and endogenous consideration of an "insistent" product. *International Journal of Industrial Organization*, 80:102798, 2022.

Stephen Leider and Özge Şahin. Contracts, biases, and consumption of access services. *Management Science*, 60(9):2198–2222, 2014.

Xiao Liu, Alan Montgomery, and Kannan Srinivasan. Analyzing bank overdraft fees with big data. *Marketing Science*, 37(6):855–882, 2018.

Mitchell J Lovett and Richard Staelin. The role of paid, earned, and owned media in building entertainment brands: Reminding, informing, and enhancing enjoyment. *Marketing Science*, 35 (1):142–157, 2016.

Kathleen B McDermott. The persistence of false memories in list recall. *Journal of Memory and language*, 35(2):212–230, 1996.

Nitin Mehta, Surendra Rajiv, and Kannan Srinivasan. Role of forgetting in memory-based choice decisions: A structural model. *Quantitative Marketing and Economics*, 2(2):107–140, 2004.

Global Newswire. Going over is over: Fido says goodbye to data overages with data overage protection. *GlobalNewsWire*, 2019. URL `https://www.globenewswire.com/en/news-release/2019/08/07/1898324/0/en/Going-Over-is-Over-Fido-says-goodbye-to-data-overages-with-Data-Overage-Protection.html`.

Pew Center on the States. Overdraft america: Confusion and concerns about bank practices. Report 1-10, The Pew Center on the States, 2012.

Matthias Rodemeier. Buy baits and consumer sophistication: Theory and field evidence from large-scale rebate promotions. Technical report, CAWM Discussion Paper, 2021.

Henry L Roediger and Kathleen B McDermott. Creating false memories: Remembering words not presented in lists. *Journal of experimental psychology: Learning, Memory, and Cognition*, 21 (4):803, 1995.

Navdeep S Sahni. Effect of temporal spacing between advertising exposures: Evidence from online field experiments. *Quantitative Marketing and Economics*, 13(3):203–247, 2015.

Bahar Şanlı and Elif Hobikoğlu. Development of internet banking as the innovative distribution channel and turkey example. *Procedia-Social and Behavioral Sciences*, 195:343–352, 2015.

Jesse M Shapiro. A'memory-jamming'theory of advertising. *Available at SSRN 903474*, 2006.

Victor Stango and Jonathan Zinman. Limited and varying consumer attention: Evidence from shocks to the salience of bank overdraft fees. *The Review of Financial Studies*, 27:990–1030, 2014.

Statista Research Department. Average bank overdraft fee in the united states from 1998 to 2021(in u.s. dollars). Report 1, Statista, 2022. URL https://www.statista.com/statistics/325565/average-bank-overdraft-fee-usa/.

Mitch Strohm. Digital banking survey: 76% of americans bank via mobile app—here are the most and least valuable features. *Forbes*, 2021. URL https://www.forbes.com/advisor/banking/digital-banking-survey-mobile-app-valuable-features/.

Arun Sundararajan. Nonlinear pricing of information goods. *Management science*, 50(12):1660–1673, 2004.

Sofia Berto Villas-Boas and J Miguel Villas-Boas. Learning, forgetting, and sales. *Management Science*, 54(11):1951–1960, 2008.

Meaghan Yuen. State of mobile banking in 2022: top apps, features, statistics and market trends. *Insider Intelligence*, 2022. URL https://www.insiderintelligence.com/insights/mobile-banking-market-trends/.

# A Additional Analysis and Extensions

## A.1 Time-invariant valuations

As noted in the main text (see, in particular, footnote 4 and surrounding discussion), we follow the precedent of Grubb (2015) in assuming that consumers' valuations $v_1$ and $v_2$ are temporally independent. To help us better understand the role of this assumption in our analysis, this appendix considers an alternate version of our model that lacks temporally-independent valuations. In particular, and as a simple benchmark for comparison, we now consider an opposite formulation in which consumers' valuations are the same in both periods as opposed to being independent.

Formally, we now impose $v_1 = v_2$, while maintaining that this (now common) valuation is drawn from a uniform distribution between 0 and 1. Note that the realization of $v_1$ is now fully informative of $v_2$, in contrast to our original specification in which the realization of $v_1$ has no bearing on $v_2$. Thus, with time-invariant valuations a consumer essentially learns $v_2$ at $t = 1$ instead of at $t = 2$ (as in our original model).

The following lemma characterizes consumer decision-making for a given penalty fee when consumers have time-invariant valuations.

**Lemma 2.** *For a consumer who subscribes to the service in the model with time-invariant valuations,* $d_1 = \mathrm{I}[v_1 > \phi]$*,* $d_2 = 1$*, and* $d_\tau = 0$*. Thus,* $D_1 = D_{12} = \widetilde{D}_{12} = \max\{1-\phi, 0\}$ *and* $D_2 = \widetilde{D}_2 = 1$*.*

From Lemma 2, we can see that, with time-invariant valuations, a consumer who subscribes to the service *always* consumes in the second period. Anticipating this, the consumer consumes in the first period if and only if her valuation $v_1 = v_2$ is higher than the penalty fee $\phi$. Intuitively, she knows she will consume at $t = 2$ regardless of her $t = 1$ consumption choice, which means consuming at $t = 1$ will be worth it if and only if the value the consumer derives from consumption exceeds the penalty fee that will inevitably be incurred following a choice to consume at $t = 1$. Note here that the consumer's consumption choices do not depend on her memory parameters $\alpha$ and $\widetilde{\alpha}$.

Lemma 2 also reveals that a consumer will never use consumption tracking in the model with time-invariant valuations; in effect, consumption tracking is irrelevant in this version of the model. As the next result shows, penalty fees likewise become irrelevant when we look at the full equilibrium under time-invariant valuations.

**Proposition 7.** *In the model with time-invariant valuations, the zero-penalty equilibrium necessarily arises in that* $\phi^* = 0$ *and* $p = 1$.

To understand the implication of Proposition 7 that the firm would never set a positive penalty fee under time-invariant valuations, recall from Lemma 2 that a consumer accurately perceives her consumption choices in this version of the model; that is, $\widetilde{D}_1 = D_1$, $\widetilde{D}_2 = D_2$, and most notably, $\widetilde{D}_{12} = D_{12}$, which means a consumer has accurate expectations regarding her likelihood of incurring the penalty fee. Due to the consumer's accurate perceptions, any potential revenues from penalty fees to the firm will be offset by an equal-sized reduction in a consumer's willingness to pay for a subscription. In turn, consumer's willingness to pay for a subscription with a positive penalty fee will be further diminished by the fact that the penalty fee might deter the consumer from consuming in the first period, thus preventing the consumer from realizing the full value of the service in both periods. As a result, the firm's expected profits when using a positive penalty fee must be less than the expected profits without a penalty fee.

### A.2 Symmetric Forgetting

In our primary model, consumers may be forgetful of past consumption but *not* past abstinence. Thus, when $t = 2$ arrives, a consumer who did not consume at $t = 1$ perfectly remembers her past abstinence, or put differently, she is *certain* that she did not consume at $t = 1$.

As discussed in Section 2 (see footnote 5 and surrounding discussion), this "asymmetric forgetting" formulation is compatible with empirical evidence that individuals are much more likely to forget an event that did occur than to falsely recall an event that did not occur. That said, it is still interesting to consider an alternate formulation. In this appendix, we consider such a formulation whereby consumers are *symmetrically* forgetful of past consumption as well as past abstinence. Namely, a consumer who did not consume at $t = 1$ may now be forgetful of past abstinence in that, when $t = 2$ arrives, she believes she did not consume at $t = 1$ with probability $1 - \alpha$ and that she did consume at $t=1$ with probability $\alpha$. Thus, regardless of whether the consumer consumed ($d_1=1$) or abstained ($d_1=0$) at $t=1$, she assigns probability $\alpha$ to the incorrect belief and probability $1-\alpha$ to the correct belief.

As we recall, in our original model a consumer's memory was entirely uninformative if $\alpha=1$. That is, with $\alpha=1$, her perceived likelihood of having consumed at $t=1$ was the same (in this case zero) regardless of whether she consumed or abstained. With symmetric forgetting, however, a consumer's memory of past consumption is entirely uninformative if $\alpha=\frac{1}{2}$. That is, with $\alpha=\frac{1}{2}$, her perceived likelihood of having consumed at $t=1$ is the same (in this case $\frac{1}{2}$) regardless of whether she consumed or abstained. Meanwhile, $\alpha>\frac{1}{2}$ would represent "negative memory," in which a consumer's belief regarding her past consumption is *negatively* correlated with her true consumption. To avoid such nonsensical cases, we restrict $\alpha\leq\frac{1}{2}$ in our analysis with symmetric forgetting.

The following result summarizes four consumption profiles that can arise in the symmetric forgetting model without consumption tracking (an exact mathematical characterization of a consumer's consumption decisions with symmetric forgetting is provided in Appendix B.3).

**Proposition 8.** *For a consumer who subscribes to the service in the symmetric forgetting model without consumption tracking:*

(i) [future abstinence-based avoidance] *if* $\phi\geq\frac{1}{\alpha}$, *then* $D_1=\min\left\{\frac{1+\widetilde{\alpha}^2\phi^2}{2},1\right\}$, $D_2=0$, *and* $D_{12}=\widetilde{D}_{12}=0$;

(ii) [memory-based avoidance] *if* $\frac{1}{\alpha}>\phi\geq\frac{1}{1-\alpha}$, *then* $D_1=\frac{1+\widetilde{\alpha}^2\phi^2}{2}$, $D_2=\frac{(1-\alpha\phi)(1-\widetilde{\alpha}^2\phi^2)}{2}$, *and* $D_{12}=\widetilde{D}_{12}=0$;

(iii) [unintentional accrual] *if* $\frac{1}{1-\alpha}>\phi\geq\frac{1}{1-\widetilde{\alpha}}$, *then* $D_1=\frac{1+\widetilde{\alpha}^2\phi^2}{2}$, $D_2=1-\alpha\phi-\frac{(1+\widetilde{\alpha}^2\phi^2)(1-2\alpha)\phi}{2}$, $D_{12}=\frac{(1+\widetilde{\alpha}^2\phi^2)(1-(1-\alpha)\phi)}{2}>0$, *and* $\widetilde{D}_{12}=0$;

(iv) [intentional accrual] *if* $\phi<\frac{1}{1-\widetilde{\alpha}}$, *then* $D_1=\frac{1+(1-\phi)^2}{2}$, $D_2=1-\alpha\phi-\frac{(1+(1-\phi)^2)(1-2\alpha)\phi}{2}$, $D_{12}=\frac{(1+(1-\phi)^2)(1-(1-\alpha)\phi)}{2}>0$, *and* $\widetilde{D}_{12}=\frac{(1+(1-\phi)^2)(1-(1-\widetilde{\alpha})\phi)}{2}>0$.

By comparing Proposition 8 to Proposition 1, we can see that the symmetric forgetting model may give rise to qualitatively distinct consumption profiles in comparison to our original model with asymmetric forgetting. Part (i), for instance, describes a consumption profile characterized by *future* abstinence-based avoidance of the penalty fee. Here, the penalty fee is sufficiently large to deter the consumer from consuming in the second period *even* if the consumer did not consume in the first period (and thus would not be at risk of incurring the penalty fee). As a result of such abstinence, the consumer is therefore always able to avoid the penalty fee, though here it is abstinence in the second period that guarantees penalty avoidance as opposed to the abstinence-based avoidance profile from our original model (Proposition 1, part ii), in which the consumer always abstains in the first period to avoid incurring the penalty fee. Interestingly, the assured abstinence in the second period may or may not be anticipated. If $\phi$ is sufficiently large ($\phi \geq \frac{1}{\tilde{\alpha}}$), the consumer correctly anticipates that she will abstain in the second period regardless of her first-period consumption choice and thus always consumes in the first period knowing it is her only opportunity to attain value from the service. Otherwise (i.e. if $\frac{1}{\tilde{\alpha}} > \phi > \frac{1}{\alpha}$), the consumer incorrectly believes that if she abstains in the first period then she might still consume — and thus attain value from the service — in the second period. As a result, the consumer might make the mistake of abstaining in the first period (as well as the second period), in the false hope of attaining a higher value from consumption in the second period.

Next, part (ii) describes a consumption profile characterized by memory-based avoidance of the penalty fee. Here, the consumer will have a sufficiently strong memory of first period consumption to ensure that she does not mistakenly consume in both periods. Unlike the case of future abstinence-based avoidance in part (i), however, now the consumer might consume in the second period if she abstains in the first period. That said, the consumer may still unnecessarily abstain in the second period following first-period abstinence, based on a misplaced concern that she might have consumed in the first period. The possibility of abstinence in *both* periods distinguishes the present case from the memory-based avoidance consumption profile seen in our original analysis with asymmetric forgetting (Proposition 1, part i). The present case also entails a higher probability of consumption in the first period. In turn, the consumption profiles characterized by unintentional and intentional accrual of the penalty fee described in parts (iii) and (iv) of Proposition 8 likewise feature higher probabilities of first-period consumption compared to the analogous consumption profiles from our original analysis (Proposition 1, part iii and iv).

In addition to its influence on consumers' consumption decisions, symmetric forgetting can also alter consumers' decisions of whether or not to use consumption tracking, if available. These latter effects are apparent in the following lemma, which characterizes a tracker's tracking decision in the model with symmetric forgetting.

**Lemma 3.** *In the symmetric forgetting model, a tracker chooses to track her consumption ($d_\tau = 1$) if and only if $k < k_\tau(\alpha, \phi | d_1)$, where:*

$$k_\tau(\alpha,\phi|d_1) = \begin{cases} \frac{\alpha(1-\alpha)\phi^2}{2}, & \phi \leq 1, \\ \frac{\alpha((1-\alpha)\phi^2-(\phi-1)^2)}{2}, & 1 < \phi < \frac{1}{\alpha},\ d_1 = 0, \\ \frac{1-\alpha}{2}, & \phi \geq \frac{1}{\alpha},\ d_1 = 0, \\ \frac{(1-\alpha)(\alpha\phi^2-(\phi-1)^2)}{2}, & 1 < \phi < \frac{1}{1-\alpha},\ d_1 = 1, \\ \frac{\alpha}{2}, & \phi \geq \frac{1}{1-\alpha},\ d_1 = 1. \end{cases}$$

As in our original analysis (see Lemma 1), a tracker still chooses to track her consumption if the tracking cost $k$ is below some threshold that depends on the consumer's forgetfulness $\alpha$ and the penalty fee $\phi$. Note, however, that the threshold in Lemma 3 now depends on the first period consumption choice $d_1$ as well, as the consumer may now track her consumption even if she did not consume in the first period. Naturally, a tracker in the original asymmetric forgetting model would, due to her perfect recollection of past abstinence, only consider tracking her first period consumption if she did, in fact, consume in the first period. With symmetric forgetting, however, she may now choose to track her consumption even if she abstained in the first period. In such cases, the consumer is worried that she might have consumed in the first period, while choosing to track consumption assures her that she can consume in the second period without incurring a penalty.

The next result characterizes the market equilibrium — which is much simpler than before — in the model with symmetric forgetting.

**Proposition 9.** *In the symmetric forgetting model, the zero-penalty benchmark equilibrium nec-*

*essarily arises in that* $\phi^* = 0$ *and* $p^* = 1$ *regardless of* $\alpha$, $\widetilde{\alpha}$, $k$, *and* $\lambda$*; in this equilibrium,* $D_{12} = \widetilde{D}_{12} = 1$ *and* $D_\tau = 0$*, while* $\Pi = 1$ *and* $CS = 0$*.*

Thus, with symmetric forgetting, it is never optimal for the firm to use a positive penalty fee. Note that this does not depend on the costs or availability of consumption tracking. In this way, the model with symmetric forgetting is not helpful for understanding the empirical use of penalty fees. Furthermore, since consumption tracking is irrelevant in the absence of penalty fees, the result also suggests that the model with symmetric forgetting is not helpful for understanding recent advances in consumption tracking technologies. With that said, these empirical limitations are not too concerning in light of the fact that they are based on a symmetric forgetting formulation that lacks empirical support (as discussed earlier). Rest assured, and as illustrated by our prior analysis, our original model with asymmetric forgetting does offer a useful lens for understanding the incidence of penalty fees as well as the recent proliferation of (and demand for) technologies that help consumers track their consumption as a means to avoid penalty fees.

### A.3 Heterogeneous Tracking Costs

Up to this point, we have primarily interpreted $\lambda$ as the share of consumers (or "trackers") with access to the consumption tracking technology and $1-\lambda$ as the share of consumers ("non-trackers") without access to consumption tracking. As noted in Section 4, however, we could equivalently assume that both segments have access to consumption tracking, but face different costs of using the technology where the cost for the "non-tracker" segment is presumed to be prohibitively high to ensure these consumers never actually use consumption tracking.

In this appendix, we revisit the idea that both consumer segments have access to consumption tracking, but face different costs of using the technology. Unlike before, however, we no longer presume that the tracking cost is prohibitively high for consumers in the high cost segment. Instead, we now suppose that all consumers have access to consumption tracking, where consumers in the size $\lambda$ segment (originally referred to as "trackers") face a tracking cost of $k = k^\ell$ while consumers in the size $1-\lambda$ segment (originally referred to as "non-trackers") face a tracking cost of $k = k^h$, with $0 \leq k^\ell \leq k^h$.

The ensuing analysis will help us understand the extent to which the main results from our original analysis still hold (qualitatively, and perhaps with minor modifications) under the present generalization of our model with heterogeneous tracking costs.

**Proposition 10.** *In the model with heterogeneous tracking costs, if $k^\ell$ is sufficiently large or $\alpha$ is sufficiently small (or both), then the equilibrium with consumption tracking is identical to the benchmark equilibrium without consumption tracking.*

As we recall from our original analysis, the availability of consumption tracking has no impact on equilibrium behavior if the tracking cost for consumers in the low cost segment is sufficiently high or if the level of consumer forgetfulness is sufficiently low (Proposition 3). Indeed, Proposition 10 shows that these predictions still hold, with the exception that the tracking cost for consumers in the low cost segment is now given by $k^\ell$ instead of $k$.

**Proposition 11.** *In the model with heterogeneous tracking costs, if $\lambda$ is sufficiently small and $k^h$ is sufficiently large with $\phi_{NT}^* > 0$, then $\phi^* = \phi_{NT}^*$ and $p^* = p_{NT}^*$. However, the equilibrium with consumption tracking may still differ from the associated benchmark equilibrium as $D_\tau > 0$ if $k_\tau(\widetilde{\alpha}, \phi_{NT}^*) \leq k^\ell < k_\tau(\alpha, \phi_{NT}^*)$, while only consumers with $k = k^h$ subscribe to the service if $k^\ell < k_\tau(\widetilde{\alpha}, \phi_{NT}^*)$. In both cases, $\Pi < \Pi_{NT}$ and $CS > CS_{NT}$.*

In our original analysis, it was shown that the firm will maintain its optimal penalty fee and optimal subscription price from the benchmark equilibrium without consumption tracking in the event that $\lambda$ is sufficiently small, or equivalently, the size of the high tracking cost segment is sufficiently large (Proposition 4). As Proposition 11 demonstrates, under the present generalization the firm will still maintain its optimal penalty fee and optimal subscription price from the benchmark equilibrium if $\lambda$ is sufficiently small, provided the tracking cost $k^h$ for the high cost segment is sufficiently large. As before, equilibrium behavior may still change relative to the benchmark equilibrium as consumers in the low tracking cost segment may make use of consumption tracking or they may refrain from subscribing altogether.

**Proposition 12.** *In the model with heterogeneous tracking costs, if $\lambda$ is sufficiently large and $k^\ell$ is sufficiently small with $\phi_{NT}^* > 0$, then $\phi^* = \phi_\tau(\alpha, k^\ell) \cdot \mathrm{I}\big[\alpha > 1 - \frac{(1-\widetilde{\alpha}^2)^2 \phi_\tau^2(\alpha, k^\ell) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha, k^\ell)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha, k^\ell)}\big] < \phi_{NT}^*$*

*and* $p^* = V(\phi^*|\widetilde{\alpha}, k = k^\ell) > p^*_{NT}$*. In this equilibrium,* $\Pi < \Pi_{NT}$ *and* $CS > CS_{NT}$*, while* $D_\tau = \widetilde{D}_\tau = 0$

Proposition 12 demonstrates the qualitative robustness of Proposition 5 under the present generalization with heterogeneous tracking costs. Namely, as in our original analysis, the firm will choose to reduce its (positive) penalty fee from its benchmark equilibrium value while raising its subscription price in the event that the size of the low tracking cost segment, i.e. $\lambda$, is sufficiently large and the tracking cost for these consumers is sufficiently small. As before, the availability of consumption tracking in this case leads to a decrease in profits and an increase in consumer surplus while consumers do not actually make use of consumption tracking in equilibrium. Here, it is not too surprising that the original result still holds under the present generalization considering the affected segment of consumers for whom consumption tracking was prohibitively costly (or simply lacked access) in our original model becomes negligible when $\lambda$ is sufficiently large.

**Proposition 13.** *In the model with heterogeneous tracking costs, if* $k^\ell$ *is sufficiently low, there exist a* $k'$, $k''$, $\widetilde{\alpha}'$, *and* $\widetilde{\alpha}''$ *with* $k' \leq k''$ *and* $\widetilde{\alpha}' \leq \widetilde{\alpha}''$ *(non-binding for some* $\alpha$, $\lambda$*) such that, if* $k' < k^\ell < k''$ *and* $\widetilde{\alpha}' < \widetilde{\alpha} < \widetilde{\alpha}''$ *with sufficiently large* $\lambda$*, then* $\phi^* = \phi_\tau(\alpha, k^\ell)$ *and* $p^* = \frac{5-4k^\ell(1-k^\ell)}{8}$*, while* $D_\tau = 0 < \widetilde{D}_\tau$*.*

Proposition 13 demonstrates that the unplanned nonuse equilibrium first described in Proposition 6 may still arise — with the same qualitative features — under the present generalization with heterogeneous tracking costs. Note here that the unplanned nonuse equilibrium in Proposition 13 is effectively an unplanned nonuse equilibrium from the perspective of consumers in the low tracking cost segment of size $\lambda$; as before, consumers in the high tracking cost segment will refrain from using consumption tracking in equilibrium.

Up to this point, our analysis of the model with heterogeneous tracking costs has closely resembled our original analysis. This includes our original analysis of the unplanned nonuse equilibrium originally formalized in Proposition 6 and revisited in Proposition 13. Importantly, however, Proposition 13 does not address the newfound (under our present generalization) possibility of an unplanned nonuse equilibrium from the perspective of consumers who face a relatively high tracking cost. Indeed, consideration of this possibility reveals some new and qualitatively distinct equilibria that can arise in the model with heterogeneous tracking costs.

In particular, recall from our original analysis, as formalized in Corollary 4, that the availability of consumption tracking could only lead to the following changes in cases where consumers do *not* actually use consumption tracking in equilibrium ($D_\tau = 0$):

- an increase in the optimal penalty fee ($\phi^* > \phi^*_{NT}$)
- an increase in profits ($\Pi > \Pi_{NT}$)
- a decrease in consumer surplus ($CS < CS_{NT}$)

By contrast, under the present generalization with heterogeneous tracking costs, these three effects no longer require an equilibrium in which consumers do not actually use consumption tracking. That is, the availability of consumption tracking may now lead to a higher penalty fee, higher profits, and/or lower consumer surplus even in cases where some consumers choose to use consumption tracking in equilibrium. These possibilities are illustrated by the following example, which is a slight modification of our original Example 1:

**Example 3.** *Let* $\alpha = \frac{9}{10}$, $\widetilde{\alpha} = \frac{25}{28}$, $k^\ell = 0$, *and* $k^h = \frac{13}{100}$. *Then, if* $\lambda > 0$ *is sufficiently small,* $\phi^* = \phi_\tau(\alpha, k^h) = 2 > \phi^*_{NT} = 0$ *and* $p^* = \frac{11{,}369}{20{,}000} < p^*_{NT} = 1$. *In this equilibrium,* $\Pi = \frac{23{,}209 - 11{,}840\lambda}{20{,}000} > \Pi_{NT} = 1$, $CS = \frac{-7{,}326 + 8{,}457\lambda}{20{,}000} < C_{NT} = 0$, *and* $0 < D_\tau = \frac{\lambda}{2} < \widetilde{D}_\tau = \frac{37 + 13\lambda}{100}$.

The possibility that the availability of consumption tracking may lead to a higher penalty fee, higher profits, and lower consumer surplus while simultaneously giving rise to an equilibrium in which some consumers use consumption tracking may be understood as a consequence of the fact that, under the present generalization with heterogeneous tracking costs, we may now see an unplanned nonuse equilibrium from the perspective of *high* tracking cost consumers. While the consumers still do not use consumption tracking in equilibrium, they face a sufficiently modest tracking cost for which they falsely expect to use consumption tracking. Meanwhile, the lower tracking cost consumer segment of size $\lambda$ may indeed use the technology, and with $\lambda$ sufficiently small, the effects of consumption tracking on the optimal penalty fee, on firm profits, and on consumer surplus will still correspond (at least qualitatively) to those associated with the unplanned nonuse equilibrium that applies to consumers in the high cost segment.

### A.4 Account Freezing

In this appendix, we explore the possibility that a consumer can have her account "frozen" before incurring a penalty fee. Formally, we now consider a variation of our model in which a consumer is prevented from choosing $d_2 = 1$ — in effect, having her account frozen — in cases where $d_1 = 1$ and $\phi > 0$.[19] The following result describes the potential consumption profiles that can arise in the model with this account freezing feature.

**Proposition 14.** *For a consumer who subscribes to the service in the model with account freezing:*
(i) [unrestrained use] *if* $\phi = 0$, *then* $D_1 = D_2 = D_{12} = \widetilde{D}_{12} = 1$*;*
(ii) [avoidance] *if* $\phi > 0$, *then* $D_1 = \frac{1}{2}$, $D_2 = \frac{1}{2}$, *and* $D_{12} = \widetilde{D}_{12} = 0$.

Proposition 14 is fairly straightforward. From part (i), if there is no penalty fee, account freezing is irrelevant and the consumer is free to consume in both periods. From part (ii), with a positive penalty fee account freezing would take effect following consumption in the first period. Anticipating this, a consumer consumes in the first period if and only if her $t = 1$ valuation is above average (i.e. at least $\frac{1}{2}$). Naturally, if she consumes at $t = 1$, her account is frozen, preventing her from consuming at $t = 2$; however, if she abstains at $t = 1$ she is then free to consume at $t = 2$ without penalty.

The next result describes the market equilibrium with account freezing:

**Proposition 15.** *In the model with account freezing, the zero-penalty benchmark equilibrium necessarily arises in that* $\phi^* = 0$ *and* $p^* = 1$ *regardless of* $\alpha$, $\widetilde{\alpha}$, $k$, *and* $\lambda$*; in this equilibrium,* $D_{12} = \widetilde{D}_{12} = 1$ *and* $D_\tau = 0$, *while* $\Pi = 1$ *and* $CS = 0$.

Proposition 15 shows that with an account freezing feature it is never optimal for the firm to use a positive penalty fee. To understand why, recall that with a positive penalty fee a consumer

[19] Since account freezing would not be relevant for a subscription that provides two units of consumption without penalty, here we assume that an account can only be frozen if $\phi > 0$.

would only consume *once* (and only expect to consume once), and thus never incur the penalty fee (Proposition 14, part i). Without a penalty fee, however, the consumer consumes (and expects to consume) in both periods. As a result, the consumer is willing to pay more for a subscription in the absence of a penalty fee, and since the firm would not collect any penalty fees even with a positive penalty fee, the firm will attain higher profits (through a higher subscription price) if it does not use a positive penalty fee.

While our preceding analysis described the implications of an exogenously-imposed account freezing feature, it is also worth considering whether (or under what circumstances) such a feature may be implemented in the first place. To do this, we now suppose that the potential implementation of an account freezing feature is endogenously determined. In particular, we now model a decision of whether or not to implement account freezing, which is made prior to all other decisions in the model. As for the question of *who* gets to make this decision, we consider three possibilities: (i) the firm, which seeks to maximize its expected profits, $\Pi$; (ii) consumers, who seek to maximize their *subjective* expectation of consumer surplus, i.e. $CS$ except calculated based on $\widetilde{\alpha}$ instead of $\alpha$; and (iii) a "social planner," who may be thought of as a benevolent regulator and seems to maximize total welfare, given as the sum of $\Pi$ and $CS$.

Since the zero-penalty equilibrium that arises with account freezing (Proposition 15) also often arises in the original model without account freezing, the decision-maker (whether the firm, a consumer, or the social planner) will often be indifferent regarding the potential implementation of account freezing. For this reason, our analysis of endogenous account freezing focuses on the less trivial cases where the decision-maker is *not* indifferent.

**Proposition 16.** *In the model with endogenously-implemented account freezing, in cases where the decision-maker is not indifferent:*

(i) *if the firm gets to decide, it will never implement account freezing;*

(ii) *if consumers get to decide, they will never implement account freezing;*

(iii) *if the social planner gets to decide, it will always implement account freezing.*

As Proposition 16 demonstrates, the potential endogenous implementation of an account freezing feature hinges on *who* gets to make this decision. From part (i), the firm will never choose to implement account freezing (in cases where it is not indifferent). This makes sense from the standpoint that account freezing guarantees the zero-penalty benchmark equilibrium with $\Pi = 1$. Since it is always possible for the firm to attain $\Pi = 1$ by setting its penalty fee to zero, any other equilibria that might arise without account freezing — and would thus be prevented by the implementation of account freezing — would entail higher profits.

From part (ii) of Proposition 16, consumers would also never choose to implement account freezing (again in cases where they are not indifferent). Intuitively, account freezing guarantees the realization of the zero-penalty equilibrium, in which consumers attain — and expect to attain — zero surplus as the firm sets its subscription price equal to a consumer's willingness-to-pay. Meanwhile, a consumer who does not subscribe to the service also attains — and expects to attain — zero surplus. Thus, in other equilibria that can arise without account freezing, if a consumer chooses to subscribe while expecting nonzero surplus then it must be the case that she expects *positive* surplus.[20] As a result, a consumer would never expect to benefit from — and may sometimes expect to be hurt by — the implementation of an account freezing feature.

Even though the firm and consumers would never choose to implement an account freezing feature, part (iii) of Proposition 16 implies that an account freezing feature can only *improve* total welfare — and would thus be implemented by a social planner (or regulator) who seeks to maximize total welfare. To understand why, recall that the implementation of account freezing guarantees the zero-penalty equilibrium in which all consumers consume in both periods while never using — or incurring a cost from using — consumption tracking. Meanwhile, total welfare is maximized when consumers consume (thus deriving value from the service) in both periods while never incurring the cost of tracking their consumption.[21] By guaranteeing the zero-penalty equilibrium, the social planner would therefore choose to implement an account freezing feature

[20] Specifically, this can happen (in the original model without account freezing) in cases where trackers and non-trackers are both served yet differ in their initial willingness-to-pay for a subscription. In these cases, consumers in the higher willingness-to-pay segment will expect positive consumer surplus.

[21] This is true because all other determinants of the two inputs in total welfare — i.e. $\Pi$ and $CS$ — are simply transfers (whether through $p$ or $\phi$) that only affect the distribution of total welfare. Formally, this can be seen from the fact that total welfare reduces to $\Pi + CS = \mathrm{E}[d_1 v_1 + d_2 v_2 - d_\tau k]$.

(in cases where the social planner is not indifferent) as this can only lead to higher total welfare than would otherwise be attained.

The prediction that an account freezing feature could increase total welfare despite mutual opposition by the firm and consumers can be reconciled in light of consumers' potentially inaccurate expectations. Namely, in these cases an account freezing feature leads to an increase in total welfare through an increase in consumer surplus that consumers fail to anticipate. In other words, our analysis indicates that an account freezing feature can only help consumers even though consumers incorrectly expect to be hurt by such a feature.

# B Consumers' Consumption Decisions

In this appendix, we provide a detailed characterization of consumers' consumption decisions (in our main model).

## B.1 Consumption Decisions in the Benchmark Model

In the benchmark model without consumption tracking, a consumer with $d_1 = 0$ knows there is no risk of accruing the penalty fee if she consumes at $t = 2$. Thus, her probability of consuming at $t = 2$ conditional on $d_1 = 0$ is simply

$$D_2(\phi | d_1 = 0) = \Pr[d_2 = 1 | d_1 = 0] = 1. \tag{A-1}$$

If $d_1 = 1$, then at $t = 2$ the consumer believes there is $1 - \alpha$ probability that $d_1 = 1$, implying the expected penalty fee from consuming at $t = 2$ is $(1 - \alpha)\phi$. Since $v_2$ is uniformly distributed between 0 and 1, the consumer's probability of consuming at $t = 2$ conditional on $d_1 = 1$ is

$$D_2(\phi | d_1 = 1) = \Pr[v_2 \geq (1 - \alpha)\phi] = \max\{1 - (1 - \alpha)\phi, 0\}. \tag{A-2}$$

Next, we would like to compute the consumer's $t = 1$ perception of her expected $t = 2$ utility conditional on her $t = 1$ choice. If $d_1 = 0$, the consumer knows she will be free to consume at $t = 2$

without risk of incurring the penalty. This implies the following $t = 1$ expectation of her $t = 2$ utility, conditional on $d_1 = 0$:

$$\widetilde{u}_2(\phi|d_1 = 0) = \mathrm{E}[v_2] = \tfrac{1}{2}. \tag{A-3}$$

If $d_1 = 1$, the consumer expects to consume at $t = 2$ (and thus incur the penalty fee) if and only if $v_2 \geq (1-\widetilde{\alpha})\phi$. Thus, her $t = 1$ expectation of her $t = 2$ utility conditional on $d_1 = 1$ is:

$$\begin{aligned} \widetilde{u}_2(\phi|d_1 = 1) &= \Pr[v_2 \geq (1-\widetilde{\alpha})\phi] \cdot \mathrm{E}[v_2 - \phi | v_2 \geq (1-\widetilde{\alpha})\phi] \\ &= \tfrac{(1-\phi)^2 - \widetilde{\alpha}^2\phi^2}{2} \cdot \mathrm{I}\big[\phi < \tfrac{1}{1-\widetilde{\alpha}}\big]. \end{aligned} \tag{A-4}$$

Using (A-1), (A-2), (A-3), and (A-4), we can now derive the unconditional probabilities of consuming at each $t = 1,2$:

$$D_1(\phi) = \Pr[v_1 + \widetilde{u}_2(\phi|d_1 = 1) \geq \widetilde{u}_2(\phi|d_1 = 0)] = \begin{cases} \frac{\max\{1+(1-\phi)^2 - \widetilde{\alpha}^2\phi^2, 0\}}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-5}$$

$$\begin{aligned} D_2(\phi) &= D_1(\phi) \cdot D_2(\phi|d_1 = 1) + (1 - D_1(\phi)) \cdot D_2(\phi|d_1 = 0) \\ &= \begin{cases} 1 - \frac{\max\{(1+(1-\phi)^2 - \widetilde{\alpha}^2\phi^2)(1-\alpha)\phi, 0\}}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1+\max\{1-(1-\alpha)\phi, 0\}}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}, \end{cases} \end{aligned} \tag{A-6}$$

where the top expression used $\max\{1-(1-\alpha)\phi, 0\} = 1-(1-\alpha)\phi$ given $\phi < \frac{1}{1-\widetilde{\alpha}}$ since $\phi < \frac{1}{1-\widetilde{\alpha}}$ and $\widetilde{\alpha} \leq \alpha$ imply $\phi < \frac{1}{1-\alpha}$.

Next, we can express a consumer's maximum $t = 0$ willingness-to-pay for a subscription as $V(\phi|\widetilde{\alpha}) = \widetilde{u}_2(\phi|d_1 = 0) + D_1(\phi) \cdot (\mathrm{E}[v_1|v_1 > \Delta] - \Delta)$ given $\Delta \equiv \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1) \geq 0$. Since $v_1$ is uniformly distributed between 0 and 1, we can express $D_1 = 1 - \Delta$ and $\mathrm{E}[v_1|v_1 > \Delta] = \frac{1+\Delta}{2}$ for $\Delta \in [0,1]$. Substituting out $\Delta = 1 - D_1$ while noting $\widetilde{u}_2(\phi|d_1 = 0) = \frac{1}{2}$, we can then verify that a consumer's willingness-to-pay reduces to:

$$V(\phi|\widetilde{\alpha}) = \frac{1 + (D_1(\phi))^2}{2}, \tag{A-7}$$

where $\widetilde{\alpha}$ implicitly enters the right-side of (A-7) through $D_1(\phi) = \widetilde{D}_1(\phi)$. While (A-7) was derived for $\Delta \in [0,1]$, we can see that it also holds for $\Delta > 1$, in which case $D_1(\phi) = 0$, implying $V(\phi|\widetilde{\alpha}) = \mathrm{E}[v_2] = \frac{1}{2}$, as desired.

The probability of consuming in both periods — and thus accruing the penalty fee — is then:

$$D_{12}(\phi) = D_1(\phi) \cdot D_2(\phi|d_1 = 1) = \begin{cases} \frac{\max\{(1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2)(1-(1-\alpha)\phi),0\}}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{\max\{1-(1-\alpha)\phi,0\}}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-8}$$

By substituting in $\widetilde{\alpha}$ for each instance of $\alpha$ in (A-5), (A-6), and (A-8), while applying $\max\{1-(1-\widetilde{\alpha})\phi, 0\} = 0$ given $\phi \geq \frac{1}{1-\widetilde{\alpha}}$, we can also derive the perceived consumption probabilities as follows:

$$\widetilde{D}_1(\phi) = \begin{cases} \frac{\max\{1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2,0\}}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-9}$$

$$\widetilde{D}_2(\phi) = \begin{cases} 1 - \frac{\max\{(1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2)(1-\widetilde{\alpha})\phi,0\}}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}, \end{cases} \tag{A-10}$$

$$\widetilde{D}_{12}(\phi) = \begin{cases} \frac{(\max\{1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2,0\})(1-(1-\widetilde{\alpha})\phi)}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ 0, & \phi \geq \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-11}$$

## B.2 Consumption Decisions in the Model with Consumption Tracking

With consumption tracking, if $d_1 = 0$ a tracker still knows there is no risk of accruing the penalty fee if she consumes at $t = 2$. Thus, there is no incentive to incur the tracking cost $k$ to track her consumption, while her probability of consuming at $t = 2$ is still $D_2(\phi|d_1 = 0) = 1$, as in (A-1), and $\widetilde{u}_2(\phi|d_1 = 0) = \frac{1}{2}$, as in (A-3). If $d_1 = 1$ and $d_\tau = 0$, the tracker's probability of consuming at $t = 2$ is still $D_2(\phi|d_1 = 1, d_\tau = 0) = \max\{1-(1-\alpha)\phi, 0\}$, as in (A-2), and $\widetilde{u}_2(\phi|d_1 = 1, d_\tau = 0) = \frac{(1-\phi)^2-\widetilde{\alpha}^2\phi^2}{2} \cdot \mathrm{I}\left[\phi < \frac{1}{1-\widetilde{\alpha}}\right]$, as in (A-4). If $d_\tau = 1$, she will know $d_1 = 1$, in which case her probability of consuming at $t = 2$ becomes

$$D_2(\phi|d_1 = 1, d_\tau = 1) = \max\{1-\phi, 0\}, \tag{A-12}$$

and $\widetilde{u}_2(\phi|d_1=1,d_\tau=1)=\frac{(1-\phi)^2}{2}\cdot \mathrm{I}[\phi<1]-k$.

Using our above work, we can derive a more general expression for the $t=1$ consumption probability in (A-5) as follows:

$$D_1(\phi)=\begin{cases}\frac{\max\{1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2,0\}}{2}, & \phi<\frac{1}{1-\widetilde{\alpha}},\ d_\tau(\phi|\widetilde{\alpha},d_1=1)=0,\\ \frac{1}{2}, & \phi\geq\frac{1}{1-\widetilde{\alpha}},\ d_\tau(\phi|\widetilde{\alpha},d_1=1)=0,\\ \frac{1-2k+(\phi-1)^2\cdot \mathrm{I}[\phi<1]}{2}, & d_\tau(\phi|\widetilde{\alpha},d_1=1)=1.\end{cases} \tag{A-13}$$

### B.3 Consumption Decisions with Symmetric Forgetting

We now provide a more detailed characterization of consumers' consumption decisions in the model with symmetric forgetting (considered in Appendix A.2). To start, in the symmetric forgetting model without consumption tracking, a consumer's probability of consuming at $t=2$ conditional on $d_1=0$ is

$$D_2(\phi|d_1=0)=\Pr[v_2\geq\alpha\phi]=\max\{1-\alpha\phi,0\}. \tag{A-14}$$

If $d_1=1$, then at $t=2$ the consumer believes there is $1-\alpha$ probability that $d_1=1$, implying the expected penalty fee from consuming at $t=2$ is $(1-\alpha)\phi$. Since $v_2$ is uniformly distributed between 0 and 1, the consumer's probability of consuming at $t=2$ conditional on $d_1=1$ is

$$D_2(\phi|d_1=1)=\Pr[v_2\geq(1-\alpha)\phi]=\max\{1-(1-\alpha)\phi,0\}. \tag{A-15}$$

Next, we would like to compute the consumer's $t=1$ perception of her expected $t=2$ utility conditional on her $t=1$ choice. If $d_1=0$, the consumer knows she will be free to consume at $t=2$ without risk of incurring the penalty. This implies the following $t=1$ expectation of her $t=2$ utility, conditional on $d_1=0$:

$$\widetilde{u}_2(\phi|d_1=0)=\Pr[v_2\geq\widetilde{\alpha}\phi]\cdot \mathrm{E}[v_2|v_2\geq\widetilde{\alpha}\phi]=\frac{1-\widetilde{\alpha}^2\phi^2}{2}\cdot \mathrm{I}\big[\phi<\tfrac{1}{\widetilde{\alpha}}\big]. \tag{A-16}$$

If $d_1=1$, the consumer expects to consume at $t=2$ (and thus incur the penalty fee) if and only if

$v_2 \geq (1-\widetilde{\alpha})\phi$. Thus, her $t=1$ expectation of her $t=2$ utility conditional on $d_1 = 1$ is:

$$\widetilde{u}_2(\phi|d_1=1) = \Pr[v_2 \geq (1-\widetilde{\alpha})\phi]\cdot \mathrm{E}[v_2-\phi|v_2 \geq (1-\widetilde{\alpha})\phi] = \tfrac{(1-\phi)^2-\widetilde{\alpha}^2\phi^2}{2}\cdot \mathrm{I}\big[\phi < \tfrac{1}{1-\widetilde{\alpha}}\big]. \quad \text{(A-17)}$$

Using (A-14), (A-15), (A-16), and (A-17) while noting $D_1(\phi) = \Pr[v_1 + \widetilde{u}_2(\phi|d_1=1) \geq \widetilde{u}_2(\phi|d_1=0)]$ and $D_2(\phi) = D_1(\phi)\cdot D_2(\phi|d_1=1) + (1-D_1(\phi))\cdot D_2(\phi|d_1=0)$, we can now derive the unconditional probabilities of consuming at each $t=1,2$:

$$D_1(\phi) = \begin{cases} \frac{1+(1-\phi)^2}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1+\widetilde{\alpha}^2\phi^2}{2}, & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{\widetilde{\alpha}}, \\ 1, & \phi > \frac{1}{\widetilde{\alpha}}, \end{cases} \quad \text{(A-18)}$$

$$D_2(\phi) = \begin{cases} 1-\alpha\phi - \frac{(1+(1-\phi)^2)(1-2\alpha)\phi}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ 1-\alpha\phi - \frac{(1+\widetilde{\alpha}^2\phi^2)(1-2\alpha)\phi}{2}, & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{1-\alpha}, \\ \frac{(1-\alpha\phi)(1-\widetilde{\alpha}^2\phi^2)}{2}, & \frac{1}{1-\alpha} < \phi \leq \frac{1}{\alpha}, \\ 0, & \phi > \frac{1}{\alpha}. \end{cases} \quad \text{(A-19)}$$

Next, we can express a consumer's maximum $t=0$ willingness-to-pay for a subscription as $V(\phi|\widetilde{\alpha}) = \widetilde{u}_2(\phi|d_1=0) + D_1(\phi)\cdot(\mathrm{E}[v_1|v_1>\Delta]-\Delta)$ given $\Delta \equiv \widetilde{u}_2(\phi|d_1=0) - \widetilde{u}_2(\phi|d_1=1) \geq 0$. Since $v_1$ is uniformly distributed between 0 and 1, we can verify that a consumer's willingness-to-pay reduces to:

$$V(\phi|\widetilde{\alpha}) = \frac{\max\{1-\widetilde{\alpha}^2\phi^2,0\}+D_1(\phi)^2}{2} = \begin{cases} \frac{1}{2}\big(1-\widetilde{\alpha}^2\phi^2 + \big(\frac{1+(1-\phi)^2}{2}\big)^2\big), & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1}{2}\big(1-\widetilde{\alpha}^2\phi^2 + \big(\frac{1+\widetilde{\alpha}^2\phi^2}{2}\big)^2\big), & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{\widetilde{\alpha}}, \\ \frac{1}{2}, & \phi > \frac{1}{\widetilde{\alpha}}. \end{cases} \quad \text{(A-20)}$$

The probability of consuming in both periods, i.e. $D_{12}(\phi) = D_1(\phi)\cdot D_2(\phi|d_1=1)$ — and thus, the probability of accruing the penalty fee — is then:

$$D_{12}(\phi) = \begin{cases} \frac{(1+(1-\phi)^2)(1-(1-\alpha)\phi)}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ \frac{(1+\widetilde{\alpha}^2\phi^2)(1-(1-\alpha)\phi)}{2}, & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{1-\alpha}, \\ 0, & \phi > \frac{1}{1-\alpha}. \end{cases} \tag{A-21}$$

By substituting in $\widetilde{\alpha}$ for each instance of $\alpha$ in (A-18), (A-19), and (A-21), while applying $\max\{1-(1-\widetilde{\alpha})\phi, 0\} = 0$ given $\phi \geq \frac{1}{1-\widetilde{\alpha}}$, we can also derive the perceived consumption probabilities as follows:

$$\widetilde{D}_1(\phi) = \begin{cases} \frac{1+(1-\phi)^2}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ \frac{1+\widetilde{\alpha}^2\phi^2}{2}, & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{\widetilde{\alpha}}, \\ 1, & \phi > \frac{1}{\widetilde{\alpha}}. \end{cases} \tag{A-22}$$

$$\widetilde{D}_2(\phi) = \begin{cases} 1-\widetilde{\alpha}\phi - \frac{(1+(1-\phi)^2)(1-2\widetilde{\alpha})\phi}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ \frac{(1-\widetilde{\alpha}\phi)(1-\widetilde{\alpha}^2\phi^2)}{2}, & \frac{1}{1-\widetilde{\alpha}} < \phi \leq \frac{1}{\widetilde{\alpha}}, \\ 0, & \phi > \frac{1}{\widetilde{\alpha}}. \end{cases} \tag{A-23}$$

$$\widetilde{D}_{12}(\phi) = \begin{cases} \frac{(1+(1-\phi)^2)(1-(1-\widetilde{\alpha})\phi)}{2}, & \phi \leq \frac{1}{1-\widetilde{\alpha}}, \\ 0, & \phi > \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-24}$$

With consumption tracking in the symmetric forgetting model, if $d_1 = 0$ a tracker may now believe that there is a risk of accruing the penalty fee if she consumes at $t = 2$. Thus, there may now be an incentive to incur the tracking cost $k$ to track her consumption even if she did not consume at $t = 1$. If she does not track her consumption following abstinence, her probability of consuming at $t = 2$ is still $D_2(\phi|d_1 = 0, d_\tau = 0) = \max\{1-\alpha\phi, 0\}$, as in (A-14), while $\widetilde{u}_2(\phi|d_1 = 0, d_\tau = 0) = \frac{1-\widetilde{\alpha}^2\phi^2}{2} \cdot \mathrm{I}\big[\phi < \frac{1}{\widetilde{\alpha}}\big]$, as in (A-16). If she does track her consumption following abstinence, her probability of consuming at $t = 2$ is then $D_2(\phi|d_1 = 0, d_\tau = 1) = 1$, while $\widetilde{u}_2(\phi|d_1 = 0, d_\tau = 1) = \mathrm{E}[v_2] - k = \frac{1}{2} - k$. If $d_1 = 1$ and $d_\tau = 0$, the tracker's probability of consuming at $t = 2$ is still $D_2(\phi|d_1 = 1, d_\tau = 0) = \max\{1-(1-\alpha)\phi, 0\}$, as in (A-15), and $\widetilde{u}_2(\phi|d_1 = 1, d_\tau = 0) = \frac{(1-\phi)^2-\widetilde{\alpha}^2\phi^2}{2} \cdot \mathrm{I}\big[\phi < \frac{1}{1-\widetilde{\alpha}}\big]$, as in (A-17). If $d_\tau = 1$, she will know $d_1 = 1$, in which case her probability of consuming at $t = 2$ becomes $D_2(\phi|d_1 = 1, d_\tau = 1) = \max\{1-\phi, 0\}$

and $\widetilde{u}_2(\phi|d_1=1,d_\tau=1)=\frac{(1-\phi)^2}{2}\cdot\mathrm{I}[\phi<1]-k$.

Given the above work, we can express the probability of consumption in the second period following a decision to use consumption tracking as

$$D_2(\phi|\widetilde{d_\tau}=1)=\max\{1-d_1\cdot\phi,0\}. \tag{A-25}$$

In turn, the perceived $t=2$ utility given the consumer expects to use consumption tracking would be

$$\widetilde{u}_2(\phi|d_\tau=1)=\frac{(\max\{1-d_1\cdot\phi,0\})^2}{2}-k. \tag{A-26}$$

# C Proofs

## C.1 Proof of Proposition 1

*Part (i).* Multiplying through by $\phi$ and rearranging, we can see that $\alpha<\frac{\phi-1}{\phi}$ implies $1-(1-\alpha)\phi<0$ and $\phi>\frac{1}{1-\alpha}$, as well as $1-(1-\widetilde{\alpha})\phi<0$ and $\phi>\frac{1}{1-\widetilde{\alpha}}$ since $\widetilde{\alpha}\leq\alpha$. The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ then follow from (A-5), (A-6), (A-8), and (A-11).

*Part (ii).* Multiplying through by $\phi$ and rearranging, while noting $\frac{\sqrt{(\phi-1)^2+1}}{\phi}>\frac{\phi-1}{\phi}$, $\widetilde{\alpha}>\frac{\sqrt{(\phi-1)^2+1}}{\phi}$ implies $1-(1-\alpha)\phi\geq1-(1-\widetilde{\alpha})\phi>0$, $\phi<\frac{1}{1-\widetilde{\alpha}}\leq\frac{1}{1-\alpha}$, and $1+(1-\phi)^2-\widetilde{\alpha}^2\phi^2<0$. The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ follow from (A-5), (A-6), (A-8), and (A-11).

*Part (iii).* Similarly, $\widetilde{\alpha}\leq\frac{\phi-1}{\phi}<\alpha$ implies $1-(1-\alpha)\phi>0\geq1-(1-\widetilde{\alpha})\phi$ and $\frac{1}{1-\widetilde{\alpha}}\leq\phi<\frac{1}{1-\alpha}$. The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ follow from (A-5), (A-6), (A-8), and (A-11).

*Part (iv).* $\frac{\phi-1}{\phi}<\widetilde{\alpha}<\frac{\sqrt{(\phi-1)^2+1}}{\phi}$ implies $1-(1-\widetilde{\alpha})\phi>0$, $\frac{1}{1-\alpha}\geq\frac{1}{1-\widetilde{\alpha}}>\phi$, and $1+(\phi-1)^2>\widetilde{\alpha}^2\phi^2$. The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ follow from (A-5), (A-6), (A-8), and (A-11). ■

## C.2 Proof of Corollary 1

*Part (i).* From (A-5), $\frac{\partial D_1(\phi)}{\partial\widetilde{\alpha}}=-\widetilde{\alpha}\phi^2\cdot\mathrm{I}\big[\frac{\phi-1}{\phi}<\widetilde{\alpha}<\frac{\sqrt{(\phi-1)^2+1}}{\phi}\big]\leq0$.

*Part (ii).* From (A-6), $\frac{\partial D_2(\phi)}{\partial\widetilde{\alpha}}=\widetilde{\alpha}(1-\alpha)\phi^3\cdot\mathrm{I}\big[\widetilde{\alpha}>\frac{\phi-1}{\phi}\big]\geq0$.

*Part (iii).* Since $D_{12}(\phi) = D_1(\phi) \cdot D_2(\phi|d_1 = 1)$, $D_2(\phi|d_1 = 1)$ is independent of $\widetilde{\alpha}$ from (A-2), and $D_1(\phi)$ is weakly decreasing in $\widetilde{\alpha}$ from part (i), $D_{12}(\phi)$ must also be weakly decreasing in $\widetilde{\alpha}$. ■

### C.3 Proof of Proposition 2

Using (1), (2), $\mathrm{E}[v_t|d_t = 1] = 1 - \frac{D_t}{2}$ (since $v_t$ is uniformly distributed from 0 to 1), (A-8), (A-9), (A-10), and (A-11), while taking $d_\tau = D_\tau = 0$ in the benchmark model without consumption tracking, and noting $p^*(\phi) = V(\phi|\widetilde{\alpha})$, a tedious calculation reveals:

$$\Pi(\phi) = \begin{cases} \frac{5}{8}, & \alpha < \frac{\phi-1}{\phi}, \\ \frac{5}{8} + \frac{(1-(1-\alpha)\phi)\phi}{2}, & \widetilde{\alpha} \leq \frac{\phi-1}{\phi} \leq \alpha, \\ 1 - \frac{(4\widetilde{\alpha}^2+8(1-\alpha)(1-\phi)+(1-\widetilde{\alpha}^2)(3+\widetilde{\alpha}^2-4\alpha)\phi^2)\phi^2}{8}, & \frac{\phi-1}{\phi} < \widetilde{\alpha} \leq \frac{\sqrt{(\phi-1)^2+1}}{\phi}, \\ \frac{1}{2}, & \widetilde{\alpha} > \frac{\sqrt{(\phi-1)^2+1}}{\phi}, \end{cases} . \tag{A-27}$$

for $\phi > 0$ while $\Pi(0) = V(0|\widetilde{\alpha}) = 1$ for any $\alpha$ and $\widetilde{\alpha}$, which guarantees $\Pi(\phi^*_{NT}) \geq 1$. Since $\Pi(\phi) < 1$ if $\alpha < \frac{\phi-1}{\phi}$ or $\widetilde{\alpha} > \frac{\sqrt{(\phi-1)^2+1}}{\phi}$, $\alpha \geq \frac{\phi^*_{NT}-1}{\phi^*_{NT}}$ and $\widetilde{\alpha} \leq \frac{\sqrt{(\phi^*_{NT}-1)^2+1}}{\phi^*_{NT}}$ must hold if $\phi^*_{NT} \neq 0$. To compute $\phi^*_{NT}$, we will compute the profit-maximizing $\phi$ for the two remaining cases, which correspond to the middle two expressions in (A-27), and then compare the implied profits from each case.

*Case I:* $\widetilde{\alpha} \leq \frac{\phi-1}{\phi} \leq \alpha$. In this case, we can use (A-27) to verify $\Pi'(\phi) > 0$ for $\phi < \frac{1}{2(1-\alpha)}$ and $\Pi'(\phi) < 0$ for $\phi > \frac{1}{2(1-\alpha)}$. Noting that $\widetilde{\alpha} \leq \frac{\phi-1}{\phi} \leq \alpha$ can be rearranged as $\frac{1}{1-\widetilde{\alpha}} \leq \phi \leq \frac{1}{1-\alpha}$, we then see that the profit-maximizing penalty fee in case I is $\phi^*_I = \max\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\}$, while $\Pi_I = \Pi(\phi^*_I) = \frac{5}{8} + \frac{(1-(1-\alpha)\phi^*_I)\phi^*_I}{2}$.

*Case II:* $\frac{\phi-1}{\phi} < \widetilde{\alpha} < \frac{\sqrt{(\phi-1)^2+1}}{\phi}$. Using (A-27), in this case there are up to three solutions to the associated first-order condition: $\phi^*_{II} \in \{0, \phi^a_{II}, \phi^b_{II}\}$, where

$$\begin{aligned} \phi^a_{II} &= \frac{3(1-\alpha)+\sqrt{(1-\alpha)(7\alpha-3)+(1-8\alpha(1-\alpha))\widetilde{\alpha}^2+4(2-3\alpha)\widetilde{\alpha}^4+2\widetilde{\alpha}^6}}{(1-\widetilde{\alpha}^2)(3+\widetilde{\alpha}^2-4\alpha)}, \\ \phi^b_{II} &= \frac{3(1-\alpha)-\sqrt{(1-\alpha)(7\alpha-3)+(1-8\alpha(1-\alpha))\widetilde{\alpha}^2+4(2-3\alpha)\widetilde{\alpha}^4+2\widetilde{\alpha}^6}}{(1-\widetilde{\alpha}^2)(3+\widetilde{\alpha}^2-4\alpha)}. \end{aligned} \tag{A-28}$$

If $(1-\alpha)(7\alpha-3)+(1-8\alpha(1-\alpha))\widetilde{\alpha}^2+4(2-3\alpha)\widetilde{\alpha}^4+2\widetilde{\alpha}^6<0$, $\phi_{II}^a$ and $\phi_{II}^b$ are not real, while $\phi_{II}^*=0$ must hold since $\frac{\partial\Pi_{II}(\phi)}{\partial\phi}\big|_{\phi=0}=0$ and $\frac{\partial^2\Pi_{II}(\phi)}{\partial\phi^2}\big|_{\phi=0}=-2(1-\alpha)-\widetilde{\alpha}^2<0$, which ensures $\Pi_{II}(\phi)$ is maximized at $\phi=0$.

If $(1-\alpha)(7\alpha-3)+(1-8\alpha(1-\alpha))\widetilde{\alpha}^2+4(2-3\alpha)\widetilde{\alpha}^4+2\widetilde{\alpha}^6>0$, it is verifiable from (A-27) and (A-28) that $\Pi(\phi_{II}^b)\leq 1$ for all $\alpha$ and $\widetilde{\alpha}$ (with $0\leq\widetilde{\alpha}\leq\alpha\leq 1$) and that $\Pi(\phi_{II}^a)\leq 1$ for all $\widetilde{\alpha}\geq\frac{\phi-1}{\phi}$. Thus, $\phi_{II}^*>0$ implies $\phi_{II}^*=\{\max\phi:\widetilde{\alpha}>\frac{\phi-1}{\phi}\}$, which converges to $\phi_{II}^*=\frac{1}{1-\widetilde{\alpha}}$. However, $\phi=\frac{1}{1-\widetilde{\alpha}}$ implies $\widetilde{\alpha}\leq\frac{\phi-1}{\phi}$. Thus, $\phi_{II}^*>0$ implies $\Pi(\phi_{II}^*)<\Pi(\frac{1}{1-\widetilde{\alpha}})\leq\Pi(\phi_I^*)$.

We can then verify $\Pi(\phi_I^*)=\frac{5}{8}+\frac{\mathrm{I}[\widetilde{\alpha}\leq 2\alpha-1]}{8(1-\alpha)}+\frac{(\alpha-\widetilde{\alpha})\mathrm{I}[\widetilde{\alpha}>2\alpha-1]}{2(1-\widetilde{\alpha})^2}$, while $\Pi(\phi_I^*)>1$ if and only if $\alpha>\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$. Thus $\phi_{NT}^*=0$ with $\Pi_{NT}=1$ if $\alpha\leq\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$ and $\phi^*=\max\{\frac{1}{2(1-\alpha)},\frac{1}{1-\widetilde{\alpha}}\}$ with $\Pi_{NT}=\Pi(\phi_I^*)=\frac{5}{8}+\frac{\mathrm{I}[\widetilde{\alpha}\leq 2\alpha-1]}{8(1-\alpha)}+\frac{(\alpha-\widetilde{\alpha})\mathrm{I}[\widetilde{\alpha}>2\alpha-1]}{2(1-\widetilde{\alpha})^2}>0$ if $\alpha>\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$. In both cases, we can then confirm the expressions for $p_{NT}^*=V(\phi_{NT}^*|\widetilde{\alpha})$ in parts (i) and (ii) using (2) with $\mathrm{E}[v_t|d_t=1]=1-\frac{D_t}{2}$ (since $v_t$ is uniformly distributed from 0 to 1), (A-9), (A-10), and (A-11), while taking $d_\tau=D_\tau=0$ in the benchmark model without consumption tracking.

Next, consumer surplus is equal to the total surplus created by the contract (i.e. consumers' cumulative valuations of the service in period with consumption) minus profits. That is,

$$\begin{aligned}
CS &= D_1(\phi)\cdot\mathrm{E}[v_1|d_1=1]+D_2\cdot\mathrm{E}[v_2|d_2=1]-\Pi(\phi)\\
&= D_1(\phi)\cdot\mathrm{E}[v_1|d_1=1]+D_1(\phi)\cdot D_2(\phi|d_1=1)\cdot\mathrm{E}[v_2|d_1=d_2=1]\\
&\qquad +(1-D_1(\phi))D_2(\phi|d_1=0)\cdot\mathrm{E}[v_2|d_1=0,d_2=1]-\Pi(\phi) \qquad\qquad \text{(A-29)}\\
&= D_1(\phi)\big(1-\tfrac{D_1(\phi)}{2}+D_2(\phi|d_1=1)\big(1-\tfrac{D_2(\phi|d_1=1)}{2}\big)\big)+\tfrac{1-D_1(\phi)}{2}-\Pi(\phi),\\
&= \tfrac{D_1(\phi)}{2}\cdot\big(1-2D_1(\phi)-D_2(\phi|d_1=1)(D_2(\phi|d_1=1)+2(\phi-1))\big),
\end{aligned}$$

which used $\mathrm{E}[v_1|d_1=1]=1-\frac{D_1(\phi)}{2}$, $\mathrm{E}[v_2|d_1=d_2=1]=1-\frac{D_2(\phi|d_1=1)}{2}$, $D_2(\phi|d_1=0)=1$, $\mathrm{E}[v_2|d_1=0,d_2=1]=\mathrm{E}[v_2]=\frac{1}{2}$ (with $v_t$ uniformly distributed between 0 and 1), $\Pi(\phi)=V(\phi|\widetilde{\alpha})+D_1(\phi)D_2(\phi|d_1=1)\phi$, and (A-7). Using (A-2), (A-5), and the expressions for $\Pi_{NT}$ derived above, we can then verify $CS_{NT}=0$ for $\alpha\leq\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$ and $CS_{NT}=\min\{\frac{1-3\alpha}{16(1-\alpha)},\frac{\widetilde{\alpha}^2-\alpha^2}{4(1-\widetilde{\alpha})^2}\}<0$ for $\alpha>\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$. Lastly, using our above work with (A-8) and (A-11), we can verify $D_{12}=\widetilde{D}_{12}=1$ given $\alpha\leq\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$ and $D_{12}=\min\{\frac{1}{4},\frac{\alpha-\widetilde{\alpha}}{2(1-\widetilde{\alpha})}\}>0=\widetilde{D}_{12}$ given $\alpha>$

$\frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$. ■

### C.4 Proof of Lemma 1

Using (A-1) and (A-2), and given $v_2$ is uniformly distributed between 0 and 1, $u_2(\phi|d_1 = 0) = \frac{1}{2}$ and $u_2(\phi|d_1 = 1, \alpha = 0) = \frac{(1-\phi)^2 \cdot \mathrm{I}[\phi<1]}{2}$. Next, given that a tracker with $d_1 = 1$ believes, when $t = 2$ arrives, that there is a $1-\alpha$ probability that $d_1 = 1$ in fact holds, her perceived expected $t = 2$ utility conditional on $d_\tau = 1$ is $u_2(\phi|d_\tau = 1) = \alpha \cdot u_2(\phi|d_1 = 0) + (1-\alpha) \cdot u_2(\phi|d_1 = 1, \alpha = 0) - k = \frac{\alpha+(1-\alpha)(\max\{1-\phi,0\})^2}{2} - k$.

Meanwhile, if $d_\tau = 0$ the tracker with $d_1 = 1$ consumes at $t = 2$ if and only if $v_2 \geq (1-\alpha)\phi$, implying her perceived expected $t = 2$ utility conditional on $d_\tau = 0$ is $u_2(\phi|d_\tau = 0) = \Pr[v_2 \geq (1-\alpha)\phi] \cdot \mathrm{E}[v_2 - (1-\alpha)\phi | v_2 \geq (1-\alpha)\phi] = \frac{(\max\{1-(1-\alpha)\phi,0\})^2}{2}$. We can then use the above expressions to verify that $u_2(\phi|d_\tau = 1) > u_2(\phi|d_\tau = 0)$, i.e. the condition for $d_\tau = 1$ given $d_1 = 1$, is equivalent to $k < k_\tau$ with $k_\tau$ as given in (3). ■

### C.5 Proof of Proposition 3

Take $k' = \frac{\alpha}{2}$. From Lemma 1, it is then verifiable that $k > k'$ implies $k > k_\tau(a, \phi)$ for $a \in \{\alpha, \widetilde{\alpha}\}$ and $\phi \geq 0$. Therefore, $D_\tau = \widetilde{D}_\tau = 0$ must hold with $k > k'$, in which case all other decisions (i.e. the firm's contract design, consumers' subscription and consumption decisions) are unaffected by the availability of consumption tracking. Thus, the equilibrium with $k > k'$ must be the same as the benchmark equilibrium without consumption tracking.

Similarly, take $\alpha' = 2k$. From Lemma 1, it is then verifiable that $\alpha < \alpha'$ implies $k > k_\tau(a, \phi)$ for $a \in \{\alpha, \widetilde{\alpha}\}$ and $\phi \geq 0$. Therefore, $D_\tau = \widetilde{D}_\tau = 0$ must hold with $\alpha < \alpha'$, in which case all other decisions (i.e. the firm's contract design, consumers' subscription and consumption decisions) are unaffected by the availability of consumption tracking. Thus, the equilibrium with $\alpha < \alpha'$ must be the same as the benchmark equilibrium without consumption tracking. ■

### C.6 Proof of Proposition 4

Let $\Pi_0(\phi,p)$ denote the firm's expected profits from a non-tracker given $\phi$ and $p$, and $\Pi_1(\phi,p)$ denote the firm's expected profits from a tracker given $\phi$ and $p$. Noting $\Pi_0(\phi_{NT}^*,p_{NT}^*)=\Pi_{NT}$, we can express the change in profits from a given $\phi$ and $p$ relative to the case with $\phi_{NT}^*$ and $p_{NT}^*$ as

$$\Pi(\phi,p)-\Pi(\phi_{NT}^*,p_{NT}^*)=(1-\lambda)(\Pi_0(\phi,p)-\Pi_{NT})+\lambda(\Pi_1(\phi,p)-\Pi_1(\phi_{NT}^*,p_{NT}^*).$$

Since $\phi_{NT}^*$ and $p_{NT}^*$ are optimal with $\lambda=0$, $\lim_{\lambda\to 0^+}\{\Pi(\phi,p)-\Pi(\phi_{NT}^*,p_{NT}^*)\}=\Pi_0(\phi,p)-\Pi_{NT}<0$ for all $\{\phi,p\}\neq\{\phi_{NT}^*,p_{NT}^*\}$. Thus, with sufficiently small $\lambda$, $\phi^*=\phi_{NT}^*$ and $p^*=p_{NT}^*$.

If $k_\tau(\widetilde{\alpha},\phi_{NT}^*)\le k<k_\tau(\alpha,\phi_{NT}^*)$ and $\phi_{NT}^*>0$, Lemma 1 implies $\widetilde{D}_\tau(\phi_{NT}^*)=0<D_\tau(\phi_{NT}^*)$ for a tracker who subscribes to the service. Since $\widetilde{D}_\tau(\phi_{NT}^*)=0$, a tracker still has $V(\phi_{NT}^*|\widetilde{\alpha})=V(\phi_{NT}^*|\widetilde{\alpha},\lambda=0)=p_{NT}^*$. Thus, since $\phi^*=\phi_{NT}^*$ and $p^*=p_{NT}^*$ with sufficiently small $\lambda$, if $k_\tau(\widetilde{\alpha},\phi_{NT}^*)\le k<k_\tau(\alpha,\phi_{NT}^*)$ a tracker still subscribes to the service yet $D_\tau>0$.

If $k<k_\tau(\widetilde{\alpha},\phi_{NT}^*)$, Lemma 1 implies $\widetilde{D}_\tau(\phi_{NT}^*)>0$ for the tracker who subscribes to the service. Using (A-13) and $\phi_{NT}^*=\max\{\frac{1}{2(1-\alpha)},\frac{1}{1-\widetilde{\alpha}}\}$, we can then see that $D_1(\phi_{NT}^*)=\frac{1}{2}-k$ for a tracker who subscribes to the service. From (A-7), which applies for all $\lambda\in[0,1]$, the tracker then has $V(\phi_{NT}^*|\widetilde{\alpha})=\frac{1}{2}\cdot\left(1+\left(\frac{1}{2}-k\right)^2\right)=\frac{5-4k(1-k)}{8}<\frac{5}{8}=p_{NT}^*$. Thus, with sufficiently small $\lambda$, the tracker does not subscribe to the service if $k<k_\tau(\widetilde{\alpha},\phi_{NT}^*)$. ■

### C.7 Proof of Proposition 5

Suppose $\lambda=1$ and $k=0$. Then, from (3), $k<k_\tau(\alpha,\phi)$ and $k<k_\tau(\widetilde{\alpha},\phi)$ must hold for all $\phi>0$, implying $D_\tau(\phi|d_1=1)=\widetilde{D}_\tau(\phi|d_1=1)=1$ for all $\phi>0$ from Lemma 1, while $d_1=d_2=1$ given $\phi=0$. It therefore follows that the model with $\lambda=1$ and $k=0$ is strategically equivalent to the benchmark model with $\widetilde{\alpha}=\alpha=0$, and that $\phi^*=0$ and $p^*=1$ from Proposition 2.

Next, using our notation from the proof of Proposition 4 and with general $\lambda$ and $k$, while noting $\Pi_1(0,1)=1$ from our above work, we can express the change in profits from a given $\phi$ and

$p$ relative to the case with $\phi = 0$ and $p = 1$ as

$$\Pi(\phi, p) - \Pi(0,1) = (1-\lambda)(\Pi_0(\phi, p) - \Pi_0(0,1)) + \lambda(\Pi_1(\phi, p) - \Pi_1(0,1)).$$

Since $\phi^* = 0$ and $p^* = 1$ with $\lambda = 1$ and $k = 0$, $\lim_{\lambda \to 1, k \to 0^+}\{\Pi(\phi, p) - \Pi(0,1)\} = \Pi_1(\phi, p) - \Pi(0,1) < 0$ for all $\{\phi, p\} \neq \{0,1\}$. Thus, with sufficiently large $\lambda$ and sufficiently small $k$, $\phi^* = 0$ and $p^* = 1$.

From the definition of $\phi_\tau(\alpha, k)$ in (4) with (3), we can then confirm $\phi_\tau(\alpha, k) = \sqrt{\frac{2k}{\alpha(1-\alpha)}} \leq 1$ for all $k \leq \frac{\alpha(1-\alpha)}{2}$. Thus, $k < \min\{\frac{\alpha(1-\alpha)}{2}, \frac{2\widetilde{\alpha}}{1-\widetilde{\alpha}^2}\}$ implies $\frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k)} < 0$, thus ensuring $\alpha \leq 1 < 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k)}$. Given $\phi_{NT}^* > 0$ with Proposition 2, it thus follows that $\phi^* = \phi_\tau(\alpha, k) \cdot \mathrm{I}\big[\alpha > 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k)}\big] = 0 < \phi_{NT}^*$, $p^* = V(0|\widetilde{\alpha}) = 1 > \frac{5}{8} = p_{NT}^*$, $\Pi = \Pi(0,1) < \Pi_{NT}$, and $CS = 0 < CS_{NT}$ with sufficiently large $\lambda$ and sufficiently small $k$.

While the above work is sufficient to establish the result, as $k$ increases (but remaining sufficiently small), it will eventually become optimal to set $\phi^* = \phi_\tau(\alpha, k)$, where $\alpha = 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k)}$ is the threshold (decreasing in $k$) at which $\Pi(\phi_\tau(\alpha, k), V(\phi_\tau(\alpha, k)|\widetilde{\alpha})) = \Pi(0,1) = 1$. ■

### C.8 Proof of Corollary 2

Given $\widetilde{\alpha} = \alpha$, Proposition 2 implies $\phi_{NT}^* = 0$, $p_{NT}^* = 1$, $\Pi_{NT} = 1$, and $CS_{NT} = 0$.

Next, for all $\phi \in (0, \phi_\tau(\alpha, k)]$ with $\widetilde{\alpha} = \alpha$, $D_\tau = \widetilde{D}_\tau = 0$, implying $\Pi(\phi, V(\phi|\alpha)) = \Pi(\phi, V(\phi|\alpha)|\lambda = 0) < \Pi(0,1|\lambda = 0) = \Pi_{NT}$ since $\phi_{NT}^* = 0$ and $p_{NT}^* = 1$ are optimal given $\lambda = 0$.

Next, suppose $\phi > \max\{\phi_\tau(\alpha, k), 1\}$ with $\widetilde{\alpha} = \alpha$. Then $D_\tau(\phi|d_1 = 1) = 1$ and $D_2(\phi|d_1 = 1) = 0$, implying $D_{12}(\phi) = 0$. Thus, in this case, $\Pi(\phi, V(\phi|\alpha)) = V(\phi|\alpha)$. Furthermore, from (A-13), we have $D_1(\phi) = \frac{1-2k}{2}$, which implies $V(\phi|\alpha) = \frac{1}{2} \cdot \big(1 + \big(\frac{1-2k}{2}\big)^2\big) = \frac{5-4k(1-k)}{8}$ from (A-7). Thus, $\Pi(\phi, V(\phi|\alpha)) = \frac{5-4k(1-k)}{8} < 1 = \Pi(0,1)$, implying that $\phi > \max\{\phi_\tau(\alpha, k), 1\}$ cannot hold at $\phi = \phi^*$ with $\widetilde{\alpha} = \alpha$.

Next, suppose $\phi \in (\phi_\tau(\alpha, k), 1]$ with $\widetilde{\alpha} = \alpha$. Then $D_\tau(\phi|d_1 = 1) = 1$ and $D_2(\phi|d_1 = 1) = \Pr[v_2 > \phi] = 1 - \phi$. Furthermore, from (A-13), we have $D_1(\phi) = \frac{1-2k+(\phi-1)^2}{2}$, which implies $D_{12} = \frac{1-2k+(\phi-1)^2}{2} \cdot (1-\phi)$ as well as $V(\phi|\alpha) = \frac{1}{2} \cdot \big(1 + \big(\frac{1-2k+(\phi-1)^2}{2}\big)^2\big)$ from (A-7). We can then compute $\Pi(\phi, V(\phi|\alpha)) = 1 - \frac{4k(2-k-\phi^2)+\phi^2(2(2-\phi)^2+\phi^2)}{8} < 1 = \Pi(0,1)$, implying that $\phi \in (\phi_\tau(\alpha,k), 1]$ cannot hold at $\phi = \phi^*$ with $\widetilde{\alpha} = \alpha$.

Collectively, the above work establishes $\phi^* = 0$ and $p^* = 1$ for all $\lambda \in [0,1]$ given $\widetilde{\alpha} = \alpha$. Furthermore, this equilibrium entails $D_1 = D_2 = 1$ and $D_\tau = 0$, and is thus identical to the zero-penalty benchmark equilibrium. ■

### C.9 Proof of Corollary 3

From (3), $k_\tau(\widetilde{\alpha}, \phi) = 0$ for all $\phi$ given $\widetilde{\alpha} = 0$. Thus, from Lemma 1, $\widetilde{D}_\tau(\phi|\widetilde{\alpha} = 0) = 0$ for all $\phi$. Using (A-13), we can then compute

$$D_1(\phi|\widetilde{\alpha} = 0) = D_1(\phi|\widetilde{\alpha} = 0, \lambda = 0) = \frac{1 + (1-\phi)^2 \cdot \mathrm{I}[\phi < 1]}{2}.$$

Recalling that $d_\tau = 1$ given $d_t = 1$ if and only if $\phi > \phi_\tau(\alpha,k)$ with (A-2) and (A-12), we can then express $D_2(\phi|d_1 = 1) - D_2(\phi|d_1 = 1, \lambda = 0) = -\lambda(\alpha\phi - \max\{\phi - 1, 0\}) \cdot \mathrm{I}\big[\phi_\tau(\alpha,k) < \phi < \frac{1}{1-\alpha}\big] \leq 0$ since $\phi < \frac{1}{1-\alpha}$ implies $\alpha > \frac{\phi-1}{\phi}$ and thus $\alpha\phi - \max\{\phi-1,0\} > \phi - 1 - \max\{\phi-1,0\} \geq 0$. In turn, we can compute

$$\begin{aligned}
&\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)) - \Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 0) \\
&\quad = D_1(\phi|\widetilde{\alpha} = 0) \cdot (D_2(\phi|d_1 = 1) - D_2(\phi|d_1 = 1, \lambda = 0)) \cdot \phi \\
&\quad = -\tfrac{\lambda\phi(1+(1-\phi)^2 \cdot \mathrm{I}[\phi<1])(\alpha\phi - \max\{\phi-1,0\})}{2} \cdot \mathrm{I}\big[\phi_\tau(\alpha,k) < \phi < \tfrac{1}{1-\alpha}\big] \leq 0.
\end{aligned}$$

Thus, $\Pi(\phi^*, V(\phi^*|\widetilde{\alpha} = 0)) \leq \Pi(\phi^*, V(\phi^*|\widetilde{\alpha} = 0)|\lambda = 0) \leq \Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha} = 0)|\lambda = 0) = \Pi_{NT}$. It then follows that $\phi^*_{NT} = 0$ implies $\phi^* = 0$, in which case $p^* = p^*_{NT} = V(0|\widetilde{\alpha} = 0) = 1$, $\Pi = \Pi_{NT} = 1$, and $CS = CS_{NT} = 0$ since $\Pi(0, V(0|\widetilde{\alpha} = 0)) = \Pi(0, V(0|\widetilde{\alpha} = 0)|\lambda = 0)$; $0 < \phi^*_{NT} < \phi_\tau(\alpha,k)$ similarly implies $\phi^* = \phi^*_{NT}$, $p^* = p^*_{NT}$, $\Pi = \Pi_{NT}$, and $CS = CS_{NT}$ since $\Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha} = 0)) = \Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha} = 0)|\lambda = 0)$ as well as $\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 0) \geq \Pi(\phi, V(\phi|\widetilde{\alpha} = 0))$ for all $\phi$ given $\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)) = \lambda\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 1) + (1-\lambda)\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 0)$ with $\Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 0) = V(\phi|\widetilde{\alpha} = 0) + D_{12}(\phi|\widetilde{\alpha} = 0, \lambda = 0) \cdot \phi \geq V(\phi|\widetilde{\alpha} = 0) + D_{12}(\phi|\widetilde{\alpha} = 0, \lambda = 1) \cdot \phi = \Pi(\phi, V(\phi|\widetilde{\alpha} = 0)|\lambda = 1)$.

Thus, $\phi^* \neq \phi^*_{NT}$ can only hold if $\phi^*_{NT} > \phi_\tau(\alpha,k)$. From Proposition 2, $\phi^*_{NT} > 0$ with $\widetilde{\alpha} = 0$ implies $\phi^*_{NT} = \frac{1}{2(1-\alpha)} > 1$ since $\phi^*_{NT} > 0$ requires $\alpha > \frac{2}{3}$ and $\alpha > \frac{2}{3}$ implies $\frac{1}{2(1-\alpha)} > 1 = \frac{1}{1-\widetilde{\alpha}}$ given $\widetilde{\alpha} = 0$. From our above work, it follows that $D_1(\phi|\phi \geq \phi^*_{NT} > 1) = \frac{1}{2}$, while $D_2(\phi|\phi \geq \phi^*_{NT} > \phi_\tau(\alpha,k), \lambda = 0, d_1 = 1) = D_{12}(\phi|\phi \geq \phi^*_{NT} > \phi_\tau(\alpha,k), \lambda = 1) = 0$. Thus, using (A-7)

$$\begin{aligned}
&\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi > \phi^*_{NT} > \phi_\tau(\alpha,k)) \\
&\quad = (1-\lambda)\cdot\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi > \phi^*_{NT} > \phi_\tau(\alpha,k), \lambda = 0) + \lambda\cdot V(\phi|\widetilde{\alpha}=0) \\
&\quad = (1-\lambda)\cdot\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi > \phi^*_{NT} > \phi_\tau(\alpha,k), \lambda = 0) + \frac{5\lambda}{8} \\
&\quad < (1-\lambda)\cdot\Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)|\lambda = 0) + \frac{5\lambda}{8} \\
&\quad = \Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)).
\end{aligned}$$

It therefore follows that $\phi^* \geq \phi^*_{NT}$ given $\phi^*_{NT} > 0$ and $\widetilde{\alpha} = 0$. From (A-13), we can then see that $D_1(\phi|\widetilde{\alpha}=0)$ is weakly decreasing in $\phi$, ensuring $D_1(\phi^*|\widetilde{\alpha}=0) \geq D_1(\phi^*_{NT}|\widetilde{\alpha}=0)$ given $\phi^* < \phi^*_{NT}$. From (A-7), $V(\phi)$ increases with $D_1(\phi)$, ensuring $p^* = V(\phi^*|\widetilde{\alpha}=0) \geq V(\phi^*_{NT}|\widetilde{\alpha}=0) = p^*_{NT}$ given $\phi^* < \phi^*_{NT}$.

Now suppose $\phi_\tau(\alpha,k) < \phi^* < \phi^*_{NT}$. Since $\Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)|\lambda=0) > \Pi(\phi^*, V(\phi^*|\widetilde{\alpha}=0)|\lambda=0)$ and $\Pi(\phi^*, V(\phi^*|\widetilde{\alpha}=0)) = (1-\lambda)\Pi(\phi^*, V(\phi^*|\widetilde{\alpha}=0)|\lambda=0) + \lambda(V(\phi^*|\widetilde{\alpha}=0) + D_{12}(\phi^*|\widetilde{\alpha}=0, \lambda=1)\phi^*) > \Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)) = (1-\lambda)\Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)|\lambda=0) + \lambda(V(\phi^*_{NT}|\widetilde{\alpha}=0) + D_{12}(\phi^*_{NT}|\widetilde{\alpha}=0, \lambda=1)\phi^*_{NT})$, it must be the case, from (A-7) and (A-13) that $D_1(\phi^*|\widetilde{\alpha}=0) > D_1(\phi^*_{NT}|\widetilde{\alpha}=0) = \frac{1}{2}$, which requires $\phi^* < 1$ given $\widetilde{\alpha} = 0$ and $\phi_\tau(\alpha,k) < \phi^*$. We can then use (A-6), (A-7), (A-12), (A-13) to compute $\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi_\tau(\alpha,k) < \phi < 1) = \frac{1}{2} + \frac{(1+(1-\phi)^2)(1+(1+\phi)^2-4(1-(1-\lambda)\alpha)\phi^2)}{8}$. We can then confirm $\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi_\tau(\alpha,k) < \phi < 1) > \Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)) = \frac{6-5\alpha-\lambda}{8(1-\alpha)}$ if and only if $\lambda > \frac{4\alpha^2((1-\phi)^2+1)\phi^2+(1-3\alpha)\phi^4+(1-2\alpha)(2(2-\phi)^2\phi^2)+3\alpha-2}{1-4(1-\alpha)\alpha\phi^2((1-\phi)^2+1)}$ and $\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi_\tau(\alpha,k) < \phi < 1) > \Pi(0,1) = 1$ holds if and only if $\lambda < 1 - \frac{\phi^2+2(2-\phi)^2}{4\alpha((1-\phi)^2+1)}$. We can then algebraically verify that $\frac{4\alpha^2((1-\phi)^2+1)\phi^2+(1-3\alpha)\phi^4+(1-2\alpha)(2(2-\phi)^2\phi^2)+3\alpha-2}{1-4(1-\alpha)\alpha\phi^2((1-\phi)^2+1)} > 1 - \frac{\phi^2+2(2-\phi)^2}{4\alpha((1-\phi)^2+1)} < 1$ for all $\Pi(\phi, V(\phi|\widetilde{\alpha}=0)|\phi_\tau(\alpha,k) < \phi < 1) < \max\{\Pi(\phi^*_{NT}, V(\phi^*_{NT}|\widetilde{\alpha}=0)), \Pi(0,1)\}$. Thus, $\phi_\tau(\alpha,k) < \phi^* < \phi^*_{NT}$ cannot hold. As a result, $\phi^* \leq \phi_\tau(\alpha,k)$ must hold if $\phi^* < \phi^*_{NT}$.

In cases with $\phi^* \leq \phi_\tau(\alpha,k) < \phi^*_{NT} = \frac{1}{2(1-\alpha)}$ (recalling $\phi_\tau(\alpha,k) < \phi^*_{NT}$ must hold given $\phi^* <$

$\phi_{NT}^{*}$), we know $D_{\tau}(\phi^{*}) = \widetilde{D}(\phi^{*}) = 0$. In turn, $CS(\phi^{*}, V(\phi^{*}|\widetilde{\alpha} = 0))$ is then given by (A-29). Using (A-29) with (A-5) and (A-2), we can then verify

$$\frac{\partial CS\left(\phi|\phi \leq \phi_{\tau}(\alpha,k) < \frac{1}{2(1-\alpha)}, \widetilde{\alpha}=0\right)}{\partial \phi} = \begin{cases} -\frac{\alpha^{2}\phi(\phi+2(1-\alpha)^{2}}{2}, & \phi \leq 1, \\ -\frac{1-(1-\alpha^{2})\phi}{2}, & \phi > 1. \end{cases}$$

Thus, $CS(\phi|\phi \leq \phi_{\tau}(\alpha,k) < \frac{1}{2(1-\alpha)}, \widetilde{\alpha} = 0)$ must decrease with $\phi$. With our above work, this establishes $CS(\phi^{*}) > CS_{NT}$ in cases with $\phi^{*} \leq \phi_{\tau}(\alpha,k) < \phi_{NT}^{*}$ given $\widetilde{\alpha} = 0$ as well as $CS(\phi^{*}) \geq CS_{NT}$ in all cases with $\widetilde{\alpha} = 0$ more broadly.

■

### C.10 Proof of Proposition 6

It suffices to demonstrate one such combination of $k_L$, $k_H$, $\widetilde{\alpha}_L$, and $\widetilde{\alpha}_H$ (not necessarily the largest possible range) for which $\phi^{*} = \phi_{\tau}(\alpha,k)$ and $p^{*} = \frac{5-4k(1-k)}{8}$, while $D_{\tau} = 0 < \widetilde{D}_{\tau}$ in equilibrium. For this purpose, let $k_L(\varepsilon) = \left(\frac{13}{100} - \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, $k_H(\varepsilon) = \left(\frac{13}{100} + \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, $\widetilde{\alpha}_L(\varepsilon) = \left(\frac{25}{28} - \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, and $\widetilde{\alpha}_H(\varepsilon) = \left(\frac{25}{28} + \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$ with $\varepsilon > 0$. Then $k_L(\varepsilon) < k_H(\varepsilon)$ and $\widetilde{\alpha}_L(\varepsilon) < \widetilde{\alpha}_H(\varepsilon)$ clearly hold for $\alpha = \frac{9}{10}$ and $\lambda = 1$. From Example 1, we can then see that $\phi^{*} = \phi_{\tau}(\alpha,k)$, $p^{*} = \frac{5-4k(1-k)}{8}$, and $D_{\tau} = 0 < \widetilde{D}_{\tau}$ in equilibrium given $k = \frac{13}{100} \in \left(k_L(\varepsilon), k_H(\varepsilon)\right)$, $\widetilde{\alpha} = \frac{25}{28} \in \left(\widetilde{\alpha}_L(\varepsilon), \widetilde{\alpha}_H(\varepsilon)\right)$, $\alpha = \frac{9}{10}$, and $\lambda = 1$. Following the same steps (provided in Appendix D.1) used to derive the solution to Example 1, it is then readily verifiable that $\phi^{*} = \phi_{\tau}(\alpha,k)$, $p^{*} = \frac{5-4k(1-k)}{8}$, and $D_{\tau} = 0 < \widetilde{D}_{\tau}$ would hold for all $k \in \left(k_L(\varepsilon), k_H(\varepsilon)\right)$ and $\widetilde{\alpha} \in \left(\widetilde{\alpha}_L(\varepsilon), \widetilde{\alpha}_H(\varepsilon)\right)$ with $\alpha = \frac{9}{10}$, $\lambda = 1$, and sufficiently small $\varepsilon > 0$. This establishes the desired result. ■

### C.11 Proof of Corollary 4

*Part (i).* Suppose $D_{\tau} = \widetilde{D}_{\tau} = 0$ with $\phi^{*} > \phi_{NT}^{*}$. Thus, from Lemma 1, $k \geq k_{\tau}(\phi^{*}, \alpha)$ and $k \geq k_{\tau}(\phi^{*}, \widetilde{\alpha})$. Since $k_{\tau}$ is weakly increasing in $\phi$, $k \geq k_{\tau}(\phi_{NT}^{*}, \alpha)$ and $k \geq k_{\tau}(\phi_{NT}^{*}, \widetilde{\alpha})$ must also hold. This implies $\Pi(\phi_{NT}^{*}) = \Pi_0(\phi_{NT}^{*}) = \Pi_{NT}$ as well as $\Pi(\phi^{*}) = \Pi_0(\phi^{*})$. Given $\phi_{NT}^{*}$ is optimal with

$\lambda = 0$, $\Pi_{NT} > \Pi_0(\phi^*)$. This implies $\Pi(\phi^*_{NT}) > \Pi(\phi^*)$, which contradicts the optimality of $\phi^*$. Thus, $\phi^* > \phi^*_{NT}$ cannot hold with $D_\tau = \widetilde{D}_\tau = 0$.

Now suppose $D_\tau > 0 = \widetilde{D}_\tau$ with $\phi^* > \phi^*_{NT} = 0$. Then $\Pi_1(\phi^*) = p^* + D_1(\phi^*) \cdot \max\{1 - \phi^*, 0\} \cdot \phi^* \leq p^* + D_1(\phi^*) \cdot \max\{1 - (1-\alpha)\phi^*, 0\} \cdot \phi^* = \Pi_0(\phi^*)$ with $p^* = V(\phi^*|\widetilde{\alpha}, d_\tau = 0)$. Thus, $\Pi(\phi^*) = (1-\lambda)\Pi_0(\phi^*) + \lambda\Pi_1(\phi^*) \leq \Pi_0(\phi^*)$. In turn, $\Pi_0(\phi^*) < \Pi_0(0) = 1$ since $\phi^*_{NT} = 0$. However, this implies $\Pi(0) = 1 > \Pi(\phi^*)$, contradicting the optimality of $\phi^*$. Thus, $\phi^* > \phi^*_{NT} = 0$ cannot hold with $D_\tau > 0 = \widetilde{D}_\tau$.

Now suppose $D_\tau > 0 = \widetilde{D}_\tau$ with $\phi^* > \phi^*_{NT} > 0$. From Proposition 2, $\phi^*_{NT} > 0$ implies $\phi^*_{NT} = \max\left\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\right\} > 1$. With $\widetilde{D}_\tau = 0$, it follows that $\Pi_1(\phi^*) = p^* + D_1(\phi^*) \cdot \max\{1 - \phi^*, 0\} \cdot \phi^* = p^* = V(\phi^*|\widetilde{\alpha}, d_\tau = 0)$. From (A-5) and (A-7) with Proposition 2, while noting $\widetilde{D}_\tau = 0$ must hold with $\phi = \phi^*_{NT}$ since $k > k_\tau(\phi^*, \widetilde{\alpha})$ implies $k > k_\tau(\phi^*_{NT}, \widetilde{\alpha})$ given $\phi^* > \phi^*_{NT}$, we can then compute $V(\phi^*|\widetilde{\alpha}) = \frac{5}{8} = V(\phi^*_{NT}|\widetilde{\alpha})$. This implies $\Pi(\phi^*) = (1-\lambda) \cdot \Pi_0(\phi^*) + \lambda \cdot \frac{5}{8}$ and $\Pi(\phi^*_{NT}) = (1-\lambda) \cdot \Pi_0(\phi^*_{NT}) + \lambda \cdot \frac{5}{8} = \Pi_{NT} + \lambda \cdot \frac{5}{8}$. Given $\phi^*_{NT}$ is optimal with $\lambda = 0$, this implies $\Pi(\phi^*_{NT}) - \Pi(\phi^*) = (1-\lambda)(\Pi_{NT} - \Pi_0(\phi^*)) > 0$, contradicting the optimality of $\phi^*$. Thus, $\phi^* > \phi^*_{NT} > 0$ cannot hold with $D_\tau > 0 = \widetilde{D}_\tau$.

Now suppose $D_\tau > 0$ and $\widetilde{D}_\tau > 0$ with $\phi^* > \phi^*_{NT}$. In this case, Proposition 2 implies $\Pi_{NT} \geq 1$. Furthermore, due to the optimality of $\phi^*_{NT}$ with $\lambda = 0$, $\Pi_0(\phi^*) < \Pi_{NT}$. Meanwhile, (A-7), (A-12), and (A-13) with $D_\tau > 0$ and $\widetilde{D}_\tau > 0$ imply $D_1(\phi^*|\lambda = 1) = \frac{1-2k+(\max\{1-\phi,0\})^2}{2}$, $V(\phi^*|\widetilde{\alpha}, \lambda = 1) = \frac{1}{2} + \frac{(1-2k+(\max\{1-\phi,0\})^2)^2}{8}$, and $D_2(\phi^*|d_1 = 1, \lambda = 1) = \max\{1 - \phi^*, 0\}$. It then follows that $\Pi_1(\phi^*|\phi^* \geq 1) \leq \frac{5-4k(1-k)}{8} < 1 \leq \Pi_{NT}$ and $\Pi_1(\phi^*|\phi^* < 1) \leq \frac{1}{2}(1 + (1-k)^2 + k\phi^2) - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} \leq \frac{1}{2}(1 + (1-k)^2 + k) - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} \leq 1 - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} < 1 \leq \Pi_{NT}$, where the third-to-last inequality used $(1-k)^2 + k \leq 1$ for all $k \in \left[0, \frac{1}{2}\right]$ (noting $k < \frac{1}{2}$ must hold from (3)) given $D_\tau > 0$ and $\widetilde{D}_\tau > 0$. At this point, we have established $\Pi_0(\phi^*) < \Pi_{NT}$ and $\Pi_1(\phi^*) < \Pi_{NT}$ given $D_\tau > 0$ and $\widetilde{D}_\tau > 0$ with $\phi^* > \phi^*_{NT}$. This implies $\Pi(\phi^*) = (1-\lambda)\Pi_0(\phi^*) + \lambda\Pi_1(\phi^*) < \Pi_{NT}$, contradicting the optimality of $\phi^*$. Thus, $\phi^* > \phi^*_{NT}$ cannot hold with $D_\tau > 0$ and $\widetilde{D}_\tau > 0$.

Collectively, the above work implies that $\phi^* > \phi^*_{NT}$ can only hold if $D_\tau = 0 < \widetilde{D}_\tau$.

*Part (ii).* The optimality of $\phi^*_{NT}$ given $\lambda = 0$ implies $\Pi_0(\phi, p) < \Pi_{NT}$ for all $(\phi, p) \neq (\phi^*_{NT}, p^*_{NT})$. Since $\Pi(\phi, p) = (1-\lambda)\Pi_0(\phi, p) + \lambda\Pi_1(\phi, p)$, $\Pi(\phi, p) > \Pi_{NT}$ can only hold if $\Pi_1(\phi, p) > \Pi_{NT}$.

Suppose $D_\tau = \widetilde{D}_\tau = 0$ at $\phi = \phi^*$. Then $\Pi_1(\phi^*) = \Pi_0(\phi^*) \le \Pi_{NT}$. Thus, $\Pi(\phi^*) \le \Pi_{NT}$ given $D_\tau = \widetilde{D}_\tau = 0$.

Now suppose $D_\tau > 0 = \widetilde{D}_\tau$ at $\phi = \phi^*$. Then $D_1(\phi^*|\lambda = 0) = D_1(\phi^*|\lambda = 1) = D_1(\phi^*)$, which implies $V(\phi^*|\widetilde{\alpha}, \lambda = 0) = V(\phi^*|\widetilde{\alpha}, \lambda = 1) = V(\phi^*|\widetilde{\alpha})$ from (A-7), while $D_\tau > 0$ at $\phi = \phi^*$ implies $D_2(\phi^*|\lambda = 1, d_1 = 1) = \max\{1-\phi^*, 0\} \le \max\{1-(1-\alpha)\phi^*, 0\} = D_2(\phi^*|\lambda = 0, d_1 = 1)$. Thus, $\Pi_1(\phi^*) = V(\phi^*|\widetilde{\alpha}) + D_1(\phi^*)D_2(\phi^*|\lambda = 1, d_1 = 1)\phi^* \le V(\phi^*|\widetilde{\alpha}) + D_1(\phi^*)D_2(\phi^*|\lambda = 0, d_1 = 1)\phi^* = \Pi_0(\phi^*) \le \Pi_{NT}$. Thus, $\Pi(\phi^*) = (1-\lambda)\Pi_0(\phi^*) + \lambda\Pi_1(\phi^*) \le \Pi_{NT}$ given $D_\tau > 0 = \widetilde{D}_\tau$.

Now suppose $D_\tau > 0$ and $\widetilde{D}_\tau > 0$ at $\phi = \phi^*$. From our work in part (i), it follows that $\Pi_1(\phi^*|\phi^* \ge 1) \le \frac{5-4k(1-k)}{8} < 1 \le \Pi_{NT}$ and $\Pi_1(\phi^*|\phi^* < 1) \le \frac{1}{2}(1+(1-k)^2+k\phi^2) - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} \le \frac{1}{2}(1+(1-k)^2+k) - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} \le 1 - \frac{\phi^2(\phi^2+2(2-\phi)^2)}{8} < 1 \le \Pi_{NT}$. Thus, $\Pi(\phi^*) < \Pi_{NT}$ given $D_\tau > 0$ and $\widetilde{D}_\tau > 0$.

Collectively, the above work implies that $\Pi(\phi^*) > \Pi_{NT}$ can only hold if $D_\tau = 0 < \widetilde{D}_\tau$.

*Part (iii).* Assume $D_\tau = 0 < \widetilde{D}_\tau$ does not hold at $\phi = \phi^*$. Part (i) then implies $\phi^* \le \phi^*_{NT}$. Therefore, $\phi^*_{NT} = 0$ implies $\phi^* = 0$, thus assuring $CS(\phi^*) = CS_{NT} = 0$ from Proposition 2. Thus, $CS(\phi^*) \neq CS_{NT}$ can only hold if $\phi^*_{NT} > 0$, in which case $\phi^*_{NT} = \max\left\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\right\}$ and $CS_{NT} = \min\left\{\frac{1-3\alpha}{16(1-\alpha)}, \frac{\widetilde{\alpha}^2-\alpha^2}{4(1-\widetilde{\alpha})^2}\right\} < 0$ from Proposition 2.

Suppose $D_\tau = \widetilde{D}_\tau = 0$ with $\phi^*_{NT} > 0$. We can then use (A-29) with (A-2) and (A-5) to derive:

$$CS(\phi^*) = \begin{cases} 0, & \phi^* \le \frac{1}{1-\widetilde{\alpha}},\ \widetilde{\alpha} \ge \frac{\sqrt{1+(\phi^*-1)^2}}{\phi^*}, \\ -\frac{(\alpha^2-\widetilde{\alpha}^2)(\phi^*)^2(1+(\phi^*-1)^2-\widetilde{\alpha}^2(\phi^*)^2)}{4}, & \phi^* \le \frac{1}{1-\widetilde{\alpha}},\ \widetilde{\alpha} < \frac{\sqrt{1+(\phi^*-1)^2}}{\phi^*}, \\ \frac{(1-\phi^*)^2-\alpha^2(\phi^*)^2}{4}, & \phi^* > \frac{1}{1-\widetilde{\alpha}}. \end{cases} \tag{A-30}$$

Thus, $CS(\phi^*) > CS_{NT}$ must hold if $\phi^* \le \frac{1}{1-\widetilde{\alpha}}$ and $\widetilde{\alpha} \ge \frac{\sqrt{1+(\phi^*-1)^2}}{\phi^*}$. For the case with $\phi^* \le \frac{1}{1-\widetilde{\alpha}}$ and $\widetilde{\alpha} < \frac{\sqrt{1+(\phi^*-1)^2}}{\phi^*}$, we can use (A-2), (A-5), and (A-7) to verify $\Pi(\phi^*|D_\tau = \widetilde{D}_\tau = 0, \phi^* \le \frac{1}{1-\widetilde{\alpha}}) > 1 = \Pi(0)$ requires $\phi^* > \frac{2\widetilde{\alpha}}{1-\widetilde{\alpha}^2}$. Next, we can confirm $2-3\phi+2(1-\widetilde{\alpha}^2)\phi^2 > 0$ for all $\phi \in \left(\frac{2\widetilde{\alpha}}{1-\widetilde{\alpha}^2}, \frac{1}{1-\widetilde{\alpha}}\right)$, which ensures $\frac{\partial CS(\phi^*)}{\partial \phi^*} = -\frac{(\alpha^2-\widetilde{\alpha}^2)\phi^*(2-3\phi^*+2(1-\widetilde{\alpha}^2)(\phi^*)^2)}{2} < 0$. If $\phi^* > \frac{1}{1-\widetilde{\alpha}}$, it must be the case that $\phi^*_{NT} = \frac{1}{2(1-\alpha)}$ and $\phi^* \le \frac{1}{2(1-\alpha)}$ since $\phi^* \le \phi^*_{NT}$ from part (i) and $\phi^*_{NT} = \max\left\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\right\}$ from

Proposition 2. Using (A-30), it then follows that $\frac{\partial CS(\phi^*)}{\partial \phi^*} = -\frac{1-(1-\alpha^2)\phi^*}{2} \leq -\frac{1}{2}\left(1 - \frac{1-\alpha^2}{2(1-\alpha)}\right) = -\frac{1-\alpha}{4} < 0$. Thus, with our above work, $CS(\phi^*)$ must be decreasing in $\phi^*$ in cases where $D_\tau = \widetilde{D}_\tau = 0$ and $\phi^*_{NT} > 0$, which ensures $CS(\phi^*) \geq CS_{NT}$ since $\phi^* \leq \phi^*_{NT}$.

Now suppose $D_\tau > 0 = \widetilde{D}_\tau$ with $\phi^*_{NT} > 0$. In this case, our above work establishes $CS(\phi^*|\lambda = 0) \geq CS_{NT}$. Thus, $CS(\phi^*) < CS_{NT}$ can only hold if $CS(\phi^*|\lambda = 1) < CS_{NT}$. Next, we can use (A-7), (A-12), and (A-13) to confirm that, with $\widetilde{D}_\tau = 0 < D_\tau$,

$$CS(\phi^*|\lambda = 1) = \begin{cases} -\frac{(2k+\phi^*(2(\phi^*-1)-\widetilde{\alpha}^2\phi^*))(1+(\phi^*-1)^2-\widetilde{\alpha}^2(\phi^*)^2)}{4}, & \phi^* \leq 1, \\ 0, & 1 < \phi^* \leq \frac{1}{1-\widetilde{\alpha}},\ \ \widetilde{\alpha} \geq \frac{\sqrt{1+(\phi^*-1)^2}}{\phi} \\ -\frac{(2k+(\phi^*-1)^2-\widetilde{\alpha}^2(\phi^*)^2)(1+(\phi^*-1)^2-\widetilde{\alpha}^2(\phi^*)^2)}{4}, & 1 < \phi^* \leq \frac{1}{1-\widetilde{\alpha}},\ \ \widetilde{\alpha} < \frac{\sqrt{1+(\phi^*-1)^2}}{\phi} \\ -\frac{k}{2}, & \phi^* > \frac{1}{1-\widetilde{\alpha}}. \end{cases}$$

We can then combine this expression with $CS_{NT} = \min\left\{\frac{1-3\alpha}{16(1-\alpha)}, \frac{\widetilde{\alpha}^2-\alpha^2}{4(1-\widetilde{\alpha})^2}\right\}$, $\alpha > \frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$ from Proposition 2 given $\phi^*_{NT} > 0$, and (3) while noting $k_\tau(\widetilde{\alpha}, \phi^*) \leq k < k_\tau(\alpha, \phi^*)$, to verify $CS(\phi^*|\lambda = 1) \geq CS_{NT}$.

Now suppose $D_\tau > 0$ and $\widetilde{D}_\tau > 0$ with $\phi^*_{NT} > 0$. In this case, our above work again establishes $CS(\phi^*|\lambda = 0) \geq CS_{NT}$. Thus, $CS(\phi^*) < CS_{NT}$ can only hold if $CS(\phi^*|\lambda = 1) < CS_{NT}$. Next, we can use (A-7), (A-12), and (A-13) to confirm that, with $D_\tau > 0$ and $\widetilde{D}_\tau > 0$,

$$CS(\phi^*|\lambda = 1) = \begin{cases} -\frac{(k+(\phi^*-1)\phi^*)(1+(\phi^*-1)^2)}{2}, & \phi^* \leq 1, \\ -\frac{k}{2}, & \phi^* > 1. \end{cases}$$

We can then combine this expression with $CS_{NT} = \min\left\{\frac{1-3\alpha}{16(1-\alpha)}, \frac{\widetilde{\alpha}^2-\alpha^2}{4(1-\widetilde{\alpha})^2}\right\}$, $\alpha > \frac{8+\max\{3\widetilde{\alpha}-1,0\}^2}{12}$ from Proposition 2 given $\phi^*_{NT} > 0$, and (3) while noting $k < k_\tau(\alpha, \phi^*)$, to verify $CS(\phi^*|\lambda = 1) \geq CS_{NT}$.

Collectively, the above work implies that $CS(\phi^*) > CS_{NT}$ can only hold if $D_\tau = 0 < \widetilde{D}_\tau$. ■

### C.12 Proof of Lemma 2

As in our original model, with time-invariant valuations, a consumer who subscribes to the service will always choose $d_2 = 1$ and $d_\tau = 0$ in cases with $d_1 = 0$; if $d_1 = 1$, the consumer will choose both $d_2 = 1$ and $d_\tau = 0$ if $v_2 > \phi$. If $v_1 = v_2 > \phi$, it then follows that choosing $d_1 = 1$ yields a total expected payoff of $v_1 + v_2 - \phi$ while choosing $d_1 = 0$ yields a total expected payoff of $v_2 < v_1 + v_2 - \phi$, implying the consumer will choose $d_1 = 1$ — followed by $d_\tau = 0$ and $d_2 = 1$ — if $v_1 > \phi$. If $v_1 \leq \phi$, choosing $d_1 = 1$ yields a total expected payoff of $v_1 - (\phi - v_2)\mathrm{I}[v_2 > (1-\alpha)\phi](1-d_\tau) - k \cdot d_\tau$ while choosing $d_1 = 0$ yields a total expected payoff of $v_2 = v_1 \geq v_1 - (\phi - v_2)\mathrm{I}[v_2 > (1-\alpha)\phi](1-d_\tau) - k \cdot d_\tau$, implying the consumer will choose $d_1 = 0$ — again followed by $d_\tau = 0$ and $d_2 = 1$ — if $v_1 \leq \phi$. Given $v_1$ is uniformly distributed between 0 and 1, it then follows that $D_1 = \max\{1-\phi, 0\}$. Since $d_2 = 1$ given $d_1 = 0$ and $d_2 = 1$ given $d_1 = 1$ and $v_1 > \phi$, while $v_1 > \phi$ must hold given $d_1 = 1$ since $d_1 = 1$ itself only holds if $v_1 > \phi$, we can then confirm $D_2 = \widetilde{D}_2 = 1$. Lastly, $D_1 = \max\{1-\phi, 0\}$ and $D_2 = \widetilde{D}_2 = 1$ then sure $D_{12} = \widetilde{D}_{12} = D_1 = \max\{1-\phi, 0\}$, as desired. ■

### C.13 Proof of Proposition 7

Using Lemma 2, we can derive a consumer's willingness-to-pay for a subscription for a given penalty fee as $V = \widetilde{D}_1 \cdot \mathrm{E}[v_1 | d_1 = 1] + \widetilde{D}_2 \cdot \mathrm{E}[v_2 | d_2 = 1] - k \cdot \widetilde{D}_\tau - \widetilde{D}_{12} \cdot \phi = \max\{1-\phi, 0\} \cdot \frac{\phi+1}{2} + 1 \cdot \frac{1}{2} - k \cdot 0 - \max\{1-\phi, 0\} \cdot \phi = \frac{1+\max\{1-\phi,0\}^2}{2}$, which used $\widetilde{D}_1 = D_1 = \max\{1-\phi, 0\}$, $\widetilde{D}_2 = D_2 = 1$, and $\widetilde{D}_\tau = D_\tau = 0$. Again using Lemma 2 while setting $p = V$, we can then derive the firm's maximum expected profit for a given penalty fee as $\Pi = V + D_{12} \cdot \phi = \frac{1+\max\{1-\phi,0\}^2}{2} + \max\{1-\phi, 0\} \cdot \phi = 1 - \frac{\min\{1,\phi^2\}}{2}$. We can then observe $\Pi = 1 - \frac{\min\{1,\phi^2\}}{2} < 1$ for all $\phi > 0$ and $\Pi = 1$ with $\phi = 0$. This ensures $\phi^* = 0$ with $p^* = V = \frac{1+\max\{1-0,0\}^2}{2} = 1$, as desired. ■

### C.14 Proof of Proposition 8

*Part (i).* The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ given $\phi \geq \frac{1}{\alpha}$ follow from (A-18), (A-19), (A-21), and (A-24). Note here that the first-period consumption probability reduces to $D_1 =$

$\min\{\frac{1+\widetilde{\alpha}^2\phi^2}{2},1\}$ in light of the fact that $\phi \geq \frac{1}{\alpha}$ implies $\phi \geq \frac{1}{1-\widetilde{\alpha}}$ given $0 \leq \widetilde{\alpha} \leq \alpha \leq \frac{1}{2}$ and that $\frac{1+\widetilde{\alpha}^2\phi^2}{2} > 1$ holds if and only if $\phi > \frac{1}{\widetilde{\alpha}}$.

*Part (ii).* The expressions for $D_1, D_2, D_{12}$, and $\widetilde{D}_{12}$ given $\frac{1}{1-\alpha} \leq \phi < \frac{1}{\alpha}$ again follow from (A-18), (A-19), (A-21), and (A-24), while also noting that $\frac{1}{1-\alpha} \leq \phi < \frac{1}{\alpha}$ implies $\frac{1}{1-\widetilde{\alpha}} \leq \phi < \frac{1}{\widetilde{\alpha}}$ since $\frac{1}{1-\widetilde{\alpha}} \leq \frac{1}{1-\alpha} \leq \frac{1}{\alpha} \leq \frac{1}{\widetilde{\alpha}}$ given $0 \leq \widetilde{\alpha} \leq \alpha \leq \frac{1}{2}$.

*Part (iii).* The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ given $\frac{1}{1-\widetilde{\alpha}} \leq \phi < \frac{1}{1-\alpha}$ again follow from (A-18), (A-19), (A-21), and (A-24), while also noting that $\phi < \frac{1}{1-\alpha}$ implies $\phi < \frac{1}{\widetilde{\alpha}}$ since $\frac{1}{1-\alpha} \leq \frac{1}{\widetilde{\alpha}}$ given $0 \leq \widetilde{\alpha} \leq \alpha \leq \frac{1}{2}$.

*Part (iv).* The expressions for $D_1$, $D_2$, $D_{12}$, and $\widetilde{D}_{12}$ given $\phi < \frac{1}{1-\widetilde{\alpha}}$ again follow from (A-18), (A-19), (A-21), and (A-24).

■

### C.15 Proof of Lemma 3

A consumer with $d_1 = 1$ has the same $t = 2$ beliefs in the symmetric forgetting model as in the original model with asymmetric forgetting. In light of this, the expression for $k_\tau(\alpha,\phi|d_1 = 1)$ in Lemma 3 is equivalent to, and can be derived in the same way (see proof of Lemma 1) as the expression for $k_\tau(\alpha,\phi)$ in (3).

Suppose Consumer A has $d_1 = 0$ and $\alpha = \alpha_A$, while Consumer B has $d_1 = 1$ and $\alpha = \alpha_B$. Then, in the symmetric forgetting model, Consumer A's $t = 2$ beliefs are equivalent to Consumer B's $t = 2$ beliefs if and only if $\alpha_A = 1-\alpha_B$. The expression for $k_\tau(\alpha,\phi|d_1 = 0)$ in Lemma 3 can then be confirmed by substituting $1-\alpha$ for $\alpha$ in the expression for $k_\tau(\alpha,\phi|d_1 = 1)$. ■

### C.16 Proof of Proposition 9

To prove Proposition 9, we will show that an equilibrium with $\phi^* > 0$ cannot exist. In this proof, we will often make use of the fact that the expression for $k_\tau(\alpha,\phi|d_1 = 0)$ in Lemma 3 given $0 \leq \widetilde{\alpha} \leq \alpha \leq \frac{1}{2}$ implies: $\widetilde{k}_{\tau|d_1=1} \leq \widetilde{k}_{\tau|d_1=0}$, $\widetilde{k}_{\tau|d_1=1} \leq k_{\tau|d_1=1}$, and $\widetilde{k}_{\tau|d_1=0} \leq k_{\tau|d_1=0}$.

*Case 1:* $k < \widetilde{k}_{\tau|d_1=1}$

Suppose $k < \widetilde{k}_{\tau|d_1=1}$ in equilibrium with $\phi^* > 0$. This implies $\widetilde{d}_{\tau|d_1=0} = \widetilde{d}_{\tau|d_1=1} = d_{\tau|d_1=1} = 1$. Using (A-26) with $D_1 = \Pr[v_1 > \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1)]$, while noting that a consumer's maximum $t = 0$ willingness-to-pay for a subscription would be given by $V(\phi|\widetilde{\alpha}) = \widetilde{u}_2(\phi|d_1 = 0) + D_1(\phi) \cdot (\mathrm{E}[v_1|v_1 > \Delta] - \Delta)$ for $\Delta \equiv \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1) \geq 0$ and with $v_1$ uniformly distributed between 0 and 1, we can verify that in this case a tracker's willingness-to-pay and probability of incurring the penalty fee would be given by:

$$V(\phi|\widetilde{\alpha}) = \begin{cases} \frac{5}{8} - k + \frac{(1-\phi)^2(2+(1-\phi)^2)}{8}, & \phi < 1, \\ \frac{5}{8} - k, & \phi \geq 1, \end{cases} \tag{A-31}$$

$$D_{12}(\phi) = \begin{cases} \frac{(1-\phi)(1+(1-\phi)^2)}{2}, & \phi < 1, \\ 0, & \phi \geq 1. \end{cases} \tag{A-32}$$

We can express the firm's maximum expected profits from a given tracker as

$$\Pi(\phi) = \begin{cases} 1 - k - \phi^2\left(1 - \phi + \frac{3\phi^2}{8}\right), & \phi < 1, \\ \frac{5}{8} - k, & \phi \geq 1, \end{cases} \tag{A-33}$$

which uses $\Pi(\phi) = p + D_{12}(\phi) \cdot \phi$ along with $p^*(\phi) = V(\phi)$. We can then see that $\Pi < 1$ must hold. Since the firm can always earn $\Pi = 1$ from a tracker by setting $\phi = 0$ (with $p = 1$), the firm therefore cannot earn higher profits from trackers than would be attained with $\phi = 0$ given $k < \widetilde{k}_{\tau|d_1=1}$.

*Case 2:* $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ *and* $k < k_{\tau|d_1=1}$

Suppose $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ and $k < k_{\tau|d_1=1}$ in equilibrium with $\phi^* > 0$. This implies $\widetilde{d}_{\tau|d_1=0} = 1$, $\widetilde{d}_{\tau|d_1=1} = 0$, and $d_{\tau|d_1=1} = 1$. Using (A-26) with $D_1 = \Pr[v_1 > \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1)]$, while noting that a consumer's maximum $t = 0$ willingness-to-pay for a subscription would be given by $V(\phi|\widetilde{\alpha}) = \widetilde{u}_2(\phi|d_1 = 0) + D_1(\phi) \cdot (\mathrm{E}[v_1|v_1 > \Delta] - \Delta)$ for $\Delta \equiv \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1) \geq 0$ and with $v_1$ uniformly distributed between 0 and 1, we can verify that in this case a tracker's willingness-to-pay and probability of incurring the penalty fee would be given by:

$$V(\phi|\widetilde{\alpha}) = \begin{cases} 1+\frac{k^2}{2}-\frac{\phi(2-(1-\widetilde{\alpha}^2)\phi)(4(1+k)-\phi(2-(1-\widetilde{\alpha}^2)\phi))}{8}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{5}{8}-\frac{k(1-k)}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}, \end{cases} \tag{A-34}$$

$$D_{12}(\phi) = 0. \tag{A-35}$$

Note, we do not need to consider $\phi < 1$ in the above expressions since $\phi < 1$ would imply $\widetilde{k}_{\tau|d_1=0} = \widetilde{k}_{\tau|d_1=1}$, thus precluding $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$.

Next, we can express the firm's maximum expected profits from a given tracker as

$$\Pi(\phi) = \begin{cases} 1+\frac{k^2}{2}-\frac{\phi(2-(1-\widetilde{\alpha}^2)\phi)(4(1+k)-\phi(2-(1-\widetilde{\alpha}^2)\phi))}{8}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{5}{8}-\frac{k(1-k)}{2}, & \phi \geq \frac{1}{1-\widetilde{\alpha}}, \end{cases} \tag{A-36}$$

which uses $\Pi(\phi) = p + D_{12}(\phi)\cdot\phi$ along with $p^*(\phi) = V(\phi)$.

For $\phi < \frac{1}{1-\widetilde{\alpha}}$, it is then readily verifiable that $\Pi(\phi) > 1$ requires

$$k > \phi\left(1+\frac{2-(1-\widetilde{\alpha}^2)\phi}{2}\right)+\sqrt{2\phi\left(1+\frac{2-(1-\widetilde{\alpha}^2)\phi}{2}\right)}$$

and that the threshold on the right side is increasing in $\phi$ for $\phi < \frac{1}{1-\widetilde{\alpha}^2}$ and decreasing in $\phi$ for $\phi > \frac{1}{1-\widetilde{\alpha}^2}$. Furthermore, the threshold is equal to $\frac{3}{2}$ if $\phi = 1$ and equal to $\frac{1+\widetilde{\alpha}^2}{2}+\sqrt{1+\widetilde{\alpha}^2} > \frac{3}{2}$ if $\phi = \frac{1}{1-\widetilde{\alpha}}$. Thus, $k > \frac{3}{2}$ must hold for $\Pi > 1$ in this case. However, $k < k_{\tau|d_1=1}$ with $1 < \phi < \frac{1}{1-\widetilde{\alpha}}$ requires $k < \frac{(1-\alpha)(\alpha\phi^2-(\phi-1)^2)}{2} < \frac{\alpha}{2} < \frac{1}{4} < \frac{3}{2}$. Thus, $\Pi(\phi) > 1$ cannot hold in this case with $1 < \phi < \frac{1}{1-\widetilde{\alpha}}$.

Lastly, by inspection, we can see that $\Pi(\phi) = \frac{5}{8}-\frac{k(1-k)}{2} < \frac{5}{8} < 1$ in this case given $\phi > \frac{1}{1-\widetilde{\alpha}}$. Thus, $\Pi(\phi) > 1$ cannot hold in this case with $\phi > \frac{1}{1-\widetilde{\alpha}}$.

Collectively, the above work shows that $\Pi < 1$ must hold. Since the firm can always earn $\Pi = 1$ from a tracker by setting $\phi = 0$ (with $p = 1$), the firm therefore cannot earn higher profits from trackers than would be attained with $\phi = 0$ given $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ and $k < k_{\tau|d_1=1}$.

*Case 3:* $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ *and* $k > k_{\tau|d_1=1}$

Suppose $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ and $k > k_{\tau|d_1=1}$ in equilibrium with $\phi^* > 0$. This implies $\widetilde{d}_{\tau|d_1=0} = 1$, $\widetilde{d}_{\tau|d_1=1} = 0$, and $d_{\tau|d_1=1} = 0$. Using (A-26) with $D_1 = \Pr[v_1 > \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1)]$, while noting that a consumer's maximum $t = 0$ willingness-to-pay for a subscription would be given by $V(\phi|\widetilde{\alpha}) = \widetilde{u}_2(\phi|d_1 = 0) + D_1(\phi) \cdot (\mathrm{E}[v_1|v_1 > \Delta] - \Delta)$ for $\Delta \equiv \widetilde{u}_2(\phi|d_1 = 0) - \widetilde{u}_2(\phi|d_1 = 1) \geq 0$ and with $v_1$ uniformly distributed between 0 and 1, we can verify that in this case a tracker's willingness-to-pay would be given by (A-34) while probability of incurring the penalty fee would be:

$$D_{12}(\phi) = \begin{cases} (1-(1-\alpha)\phi)\big(1+k-\phi+\frac{(1-\widetilde{\alpha}^2)\phi^2}{2}\big), & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ (1-(1-\alpha)\phi)\big(\frac{1}{2}+k\big), & \frac{1}{1-\widetilde{\alpha}} < \phi < \frac{1}{1-\alpha}, \\ 0, & \phi \geq \frac{1}{1-\alpha}. \end{cases} \tag{A-37}$$

Next, we can express the firm's maximum expected profits from a given tracker as

$$\Pi(\phi) = \begin{cases} 1-\frac{\widetilde{\alpha}^2\phi^2}{2}+\frac{(2k+(1-\widetilde{\alpha}^2)\phi^2)^2}{8}-\frac{(1-\alpha)\phi^2(2(1-\phi+k)+(1-\widetilde{\alpha}^2)\phi^2)}{2}, & \phi < \frac{1}{1-\widetilde{\alpha}}, \\ \frac{5}{8}-\frac{k(1-k)}{2}+(1-(1-\alpha)\phi)\big(\frac{1}{2}+k\big)\phi, & \frac{1}{1-\widetilde{\alpha}} \leq \phi < \frac{1}{1-\alpha}, \\ \frac{5}{8}-\frac{k(1-k)}{2}, & \phi \geq \frac{1}{1-\alpha}, \end{cases} \tag{A-38}$$

which uses $\Pi(\phi) = p + D_{12}(\phi) \cdot \phi$ along with $p^*(\phi) = V(\phi)$.

For $\phi < \frac{1}{1-\widetilde{\alpha}}$, it is readily verifiable that $\Pi(\phi) > 1$ requires

$$k > \frac{(1+\widetilde{\alpha}^2-2\alpha)\phi^2}{2} + \phi\sqrt{1+\widetilde{\alpha}^2-2\alpha+(1-(1-\alpha)\phi)^2}.$$

However, $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ cannot hold with $\phi < 1$ because $\phi < 1$ implies $\widetilde{k}_{\tau|d_1=0} = \widetilde{k}_{\tau|d_1=1}$. In turn, $k < \widetilde{k}_{\tau|d_1=0}$ with $1 < \phi < \frac{1}{1-\widetilde{\alpha}}$ requires $k < \frac{\widetilde{\alpha}((1-\widetilde{\alpha})\phi^2-(\phi-1)^2)}{2}$, which is less than the threshold shown above. Thus, $\Pi(\phi) > 1$ cannot hold in this case with $\phi < \frac{1}{1-\widetilde{\alpha}}$.

For $\frac{1}{1-\widetilde{\alpha}} < \phi < \frac{1}{1-\alpha}$, it is readily verifiable that $\Pi(\phi) > 1$ requires

$$k > \frac{3}{2} - 2\phi(1-(1-\alpha)\phi).$$

Next, we can demonstrate that this threshold is higher than $\frac{\alpha}{2}$. To see this, note that $\frac{\partial}{\partial\alpha}\left(\frac{3}{2} - 2\phi(1-(1-\alpha)\phi) - \frac{\alpha}{2}\right) = -\frac{1}{2} - 2\phi < 0$, implying this difference, i.e. $\frac{3}{2} - 2\phi(1-(1-\alpha)\phi) - \frac{\alpha}{2}$, is minimized at $\alpha = \frac{1}{2}$ given $\alpha \leq \frac{1}{2}$, in which case the difference reduces to $\frac{5}{4} - 2\phi + \phi^2 = \frac{1}{4} + (\phi-1)^2 > 0$. Thus, $\Pi(\phi) > 1$ requires $k > \frac{3}{2} - 2\phi(1-(1-\alpha)\phi) > \frac{\alpha}{2}$, but $k < \widetilde{k}_{\tau|d_1=0}$ requires $k < \frac{\alpha}{2}$. As a result, $\Pi(\phi) > 1$ cannot hold in this case with $\frac{1}{1-\widetilde{\alpha}} < \phi < \frac{1}{1-\alpha}$.

Lastly, by inspection, we can see that $\Pi(\phi) = \frac{5}{8} - \frac{k(1-k)}{2} < \frac{5}{8} < 1$ in this case given $\phi > \frac{1}{1-\alpha}$. Thus, $\Pi(\phi) > 1$ cannot hold in this case with $\phi > \frac{1}{1-\alpha}$.

Collectively, the above work shows that $\Pi < 1$ must hold in case 3. Since the firm can always earn $\Pi = 1$ from a tracker by setting $\phi = 0$ (with $p = 1$), the firm therefore cannot earn higher profits from trackers than would be attained with $\phi = 0$ given $\widetilde{k}_{\tau|d_1=1} < k < \widetilde{k}_{\tau|d_1=0}$ and $k > k_{\tau|d_1=1}$.

*Case 4: $k > \widetilde{k}_{\tau|d_1=1}$*

Note that we can break this case down into subcases depending on whether $k < k_{\tau|d_1=1}$ or $k > k_{\tau|d_1=1}$. Either way, $V(\phi|\widetilde{\alpha})$ will be the same as in the no-tracking benchmark, while $D_{12}$ will be less than or equal to its value from the no-tracking benchmark. Thus, the firm cannot increase its profits from trackers relative to the no-tracking benchmark $k > \widetilde{k}_{\tau|d_1=1}$. In conjunction with our prior work, we see that it is never possible for the firm to increase its profits from trackers relative to the no-tracking benchmark.

Since the decision problem for non-trackers' is equivalent to the decision problem for trackers in cases with $k > \max\{k_\tau, \widetilde{k}_\tau\}$, the present work (for case 4) also implies that the firm cannot increase its profits from non-trackers relative to the no-tracking benchmark. Furthermore, this will be true regardless of whether $k > \widetilde{k}_{\tau|d_1=1}$ applies to trackers. As a result, it is never possible for the firm to increase its profits from trackers relative to the no-tracking benchmark. Since it is also never possible for the firm to increase its profits from trackers, it then follows that $\phi^* = 0$ and $p^* = 1$ must hold in the symmetric forgetting model, while $\phi^* = 0$ and $p^* = 1$ imply $D_{12} = \widetilde{D}_{12} = 1$, $D_\tau = 0$, $\Pi = 1$, and $CS = 0$. ■

## C.17 Proof of Proposition 10

Take $k' = \frac{\alpha}{2}$. From Lemma 1, it is then verifiable that $k^\ell > k'$ implies $k^h > k^\ell > k_\tau(a,\phi)$ for $a \in \{\alpha, \widetilde{\alpha}\}$ and $\phi \geq 0$. Therefore, $D_\tau = \widetilde{D}_\tau = 0$ must hold with $k^\ell > k'$, in which case all other decisions (i.e. the firm's contract design, consumers' subscription and consumption decisions) are unaffected by the availability of consumption tracking. Thus, the equilibrium with $k^\ell > k'$ must be the same as the benchmark equilibrium without consumption tracking.

Similarly, take $\alpha' = 2k^\ell$. From Lemma 1, it is then verifiable that $\alpha < \alpha'$ implies $k^h > k^\ell > k_\tau(a,\phi)$ for $a \in \{\alpha, \widetilde{\alpha}\}$ and $\phi \geq 0$. Therefore, $D_\tau = \widetilde{D}_\tau = 0$ must hold with $\alpha < \alpha'$, in which case all other decisions (i.e. the firm's contract design, consumers' subscription and consumption decisions) are unaffected by the availability of consumption tracking. Thus, the equilibrium with $\alpha < \alpha'$ must be the same as the benchmark equilibrium without consumption tracking. ■

## C.18 Proof of Proposition 11

Let $\Pi_0(\phi, p)$ denote the firm's expected profits from a consumer with $k = k^h$ given $\phi$ and $p$, and $\Pi_1(\phi, p)$ denote the firm's expected profits from a consumer with $k = k^\ell$ given $\phi$ and $p$. Noting $\Pi_0(\phi^*_{NT}, p^*_{NT}) = \Pi_{NT}$, we can express the change in profits from a given $\phi$ and $p$ relative to the case with $\phi^*_{NT}$ and $p^*_{NT}$ as

$$\Pi(\phi, p) - \Pi(\phi^*_{NT}, p^*_{NT}) = (1-\lambda)(\Pi_0(\phi, p) - \Pi_{NT}) + \lambda(\Pi_1(\phi, p) - \Pi_1(\phi^*_{NT}, p^*_{NT}).$$

Since $\phi^*_{NT}$ and $p^*_{NT}$ are optimal with $\lambda = 0$ and $k^h > \frac{\alpha}{2}$ (since this ensures $k^h > k_\tau(a,\phi)$ for $a \in \{\alpha, \widetilde{\alpha}\}$ and $\phi \geq 0$), $\lim_{\lambda \to 0^+}\{\Pi(\phi, p) - \Pi(\phi^*_{NT}, p^*_{NT})\} = \Pi_0(\phi, p) - \Pi_{NT} < 0$ for all $\{\phi, p\} \neq \{\phi^*_{NT}, p^*_{NT}\}$ and $k^h$ sufficiently large. Thus, with sufficiently small $\lambda$ and sufficiently large $k^h$, $\phi^* = \phi^*_{NT}$ and $p^* = p^*_{NT}$.

If $k_\tau(\widetilde{\alpha}, \phi^*_{NT}) \leq k^\ell < k_\tau(\alpha, \phi^*_{NT})$ and $\phi^*_{NT} > 0$, Lemma 1 implies $\widetilde{D}_\tau(\phi^*_{NT}) = 0 < D_\tau(\phi^*_{NT})$ for a consumer with $k = k^\ell$ who subscribes to the service. Since $\widetilde{D}_\tau(\phi^*_{NT}) = 0$, a consumer with $k = k^\ell$ still has $V(\phi^*_{NT}|\widetilde{\alpha}) = V(\phi^*_{NT}|\widetilde{\alpha}, \lambda = 0) = p^*_{NT}$. Thus, since $\phi^* = \phi^*_{NT}$ and $p^* = p^*_{NT}$ with sufficiently small $\lambda$ and sufficiently large $k^h$, if $k_\tau(\widetilde{\alpha}, \phi^*_{NT}) \leq k^\ell < k_\tau(\alpha, \phi^*_{NT})$ a consumer with

$k = k^\ell$ still subscribes to the service yet $D_\tau > 0$.

If $k^\ell < k_\tau(\widetilde{\alpha}, \phi^*_{NT})$, Lemma 1 implies $\widetilde{D}_\tau(\phi^*_{NT}) > 0$ for the consumer with $k = k^\ell$ who subscribes to the service. Using (A-13) and $\phi^*_{NT} = \max\{\frac{1}{2(1-\alpha)}, \frac{1}{1-\widetilde{\alpha}}\}$, we can then see that $D_1(\phi^*_{NT}) = \frac{1}{2} - k^\ell$ for a consumer with $k = k^\ell$ who subscribes to the service. From (A-7), which applies for all $\lambda \in [0,1]$, the consumer then has $V(\phi^*_{NT}|\widetilde{\alpha}) = \frac{1}{2} \cdot \left(1 + \left(\frac{1}{2} - k^\ell\right)^2\right) = \frac{5-4k^\ell(1-k^\ell)}{8} < \frac{5}{8} = p^*_{NT}$. Thus, with sufficiently small $\lambda$ and sufficiently large $k^h$, the consumer with $k = k^\ell$ does not subscribe to the service if $k^\ell < k_\tau(\widetilde{\alpha}, \phi^*_{NT})$. ■

### C.19 Proof of Proposition 12

Suppose $\lambda = 1$ and $k^\ell = 0$. Then, from (3), $k^\ell < k_\tau(\alpha, \phi)$ and $k^\ell < k_\tau(\widetilde{\alpha}, \phi)$ must hold for all $\phi > 0$, implying $D_\tau(\phi|d_1 = 1) = \widetilde{D}_\tau(\phi|d_1 = 1) = 1$ for all $\phi > 0$ from Lemma 1, while $d_1 = d_2 = 1$ given $\phi = 0$. It therefore follows that the model with $\lambda = 1$ and $k^\ell = 0$ is strategically equivalent to the benchmark model with $\widetilde{\alpha} = \alpha = 0$, and that $\phi^* = 0$ and $p^* = 1$ from Proposition 2.

Next, using our notation from the proof of Proposition 4 and with general $\lambda$ and $k^\ell$, while noting $\Pi_1(0,1) = 1$ from our above work, we can express the change in profits from a given $\phi$ and $p$ relative to the case with $\phi = 0$ and $p = 1$ as

$$\Pi(\phi, p) - \Pi(0,1) = (1-\lambda)(\Pi_0(\phi, p) - \Pi_0(0,1)) + \lambda(\Pi_1(\phi, p) - \Pi_1(0,1)).$$

Since $\phi^* = 0$ and $p^* = 1$ with $\lambda = 1$ and $k^\ell = 0$, $\lim_{\lambda \to 1, k^\ell \to 0^+}\{\Pi(\phi, p) - \Pi(0,1)\} = \Pi_1(\phi, p) - \Pi(0,1) < 0$ for all $\{\phi, p\} \neq \{0,1\}$. Thus, with sufficiently large $\lambda$ and sufficiently small $k^\ell$, $\phi^* = 0$ and $p^* = 1$.

From the definition of $\phi_\tau(\alpha, k^\ell)$ in (4) with (3), we can then confirm $\phi_\tau(\alpha, k^\ell) = \sqrt{\frac{2k^\ell}{\alpha(1-\alpha)}} \leq 1$ for all $k^\ell \leq \frac{\alpha(1-\alpha)}{2}$. Thus, $k^\ell < \min\{\frac{\alpha(1-\alpha)}{2}, \frac{2\widetilde{\alpha}}{1-\widetilde{\alpha}^2}\}$ implies $\frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k^\ell) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k^\ell)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k^\ell)} < 0$, thus ensuring $\alpha \leq 1 < 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k^\ell) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k^\ell)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k^\ell)}$. Given $\phi^*_{NT} > 0$ with Proposition 2, it thus follows that $\phi^* = \phi_\tau(\alpha, k^\ell) \cdot \mathrm{I}\left[\alpha > 1 - \frac{(1-\widetilde{\alpha}^2)^2\phi_\tau^2(\alpha,k^\ell) - 4\widetilde{\alpha}^2}{8(1-\phi_\tau(\alpha,k^\ell)) + 4(1-\widetilde{\alpha}^2)\phi_\tau^2(\alpha,k^\ell)}\right] = 0 < \phi^*_{NT}$, $p^* = V(0|\widetilde{\alpha}) = 1 > \frac{5}{8} = p^*_{NT}$, $\Pi = \Pi(0,1) < \Pi_{NT}$, and $CS = 0 < CS_{NT}$ with sufficiently large $\lambda$ and sufficiently small $k^\ell$. ■

### C.20 Proof of Proposition 13

It suffices to demonstrate one such combination of $k'$, $k''$, $\widetilde{\alpha}'$, and $\widetilde{\alpha}''$ (not necessarily the largest possible range) for which $\phi^* = \phi_\tau(\alpha, k^\ell)$ and $p^* = \frac{5-4k^\ell(1-k^\ell)}{8}$, while $D_\tau = 0 < \widetilde{D}_\tau$ in equilibrium. For this purpose, let $k'(\varepsilon) = \left(\frac{13}{100} - \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, $k''(\varepsilon) = \left(\frac{13}{100} + \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, $\widetilde{\alpha}'(\varepsilon) = \left(\frac{25}{28} - \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$, and $\widetilde{\alpha}''(\varepsilon) = \left(\frac{25}{28} + \varepsilon\right) \cdot \mathrm{I}\left[\alpha = \frac{9}{10}, \lambda = 1\right]$ with $\varepsilon > 0$. Then $k'(\varepsilon) < k''(\varepsilon)$ and $\widetilde{\alpha}'(\varepsilon) < \widetilde{\alpha}''(\varepsilon)$ clearly hold for $\alpha = \frac{9}{10}$ and $\lambda = 1$. From Example 1, we can then see that $\phi^* = \phi_\tau(\alpha, k^\ell)$, $p^* = \frac{5-4k^\ell(1-k^\ell)}{8}$, and $D_\tau = 0 < \widetilde{D}_\tau$ in equilibrium given $k^\ell = \frac{13}{100} \in \left(k'(\varepsilon), k''(\varepsilon)\right)$, $\widetilde{\alpha} = \frac{25}{28} \in \left(\widetilde{\alpha}'(\varepsilon), \widetilde{\alpha}''(\varepsilon)\right)$, $\alpha = \frac{9}{10}$, and $\lambda = 1$. Following the same steps (provided in Appendix D.1) used to derive the solution to Example 1 (and also applicable to Example 3), it is then readily verifiable that $\phi^* = \phi_\tau(\alpha, k^\ell)$, $p^* = \frac{5-4k^\ell(1-k^\ell)}{8}$, and $D_\tau = 0 < \widetilde{D}_\tau$ would hold for all $k^\ell \in \left(k'(\varepsilon), k_H(\varepsilon)\right)$ and $\widetilde{\alpha} \in \left(\widetilde{\alpha}'(\varepsilon), \widetilde{\alpha}''(\varepsilon)\right)$ with $\alpha = \frac{9}{10}$, $\lambda = 1$, and sufficiently small $\varepsilon > 0$. This establishes the desired result. ■

### C.21 Proof Proposition 14

*Part (i).* If $\phi = 0$, account freezing is inactive. It then follows from our original analysis without account freezing in Appendix B that $D_1 = D_2 = D_{12} = \widetilde{D}_{12} = 1$ in this case.

*Part (ii).* If $\phi > 0$, account freezing is active. This is equivalent to taking $\phi > 0$ to be sufficiently large in our original benchmark model without account freezing (or consumption tracking), or more specifically, taking $\phi > 0$ to be large enough to ensure that the consumer never consumes and never expects to consume at $t = 2$ conditional on having consumed at $t = 1$. This corresponds to part (i) of Proposition 1, which yields $D_1 = D_2 = \frac{1}{2}$ and $D_{12} = \widetilde{D}_{12} = 0$, as desired. ■

### C.22 Proof Proposition 15

Suppose $\phi > 0$. The consumer would consume at $t = 1$ if and only if $v_1 > \frac{1}{2}$ while consuming at $t = 2$ if and only if $d_1 = 0$. The consumer's valuation of the service contract would then be $V = \Pr[v_1 > \frac{1}{2}] \cdot \mathrm{E}[v_1 | v_1 > \frac{1}{2}] + (1 - \Pr[v_1 > \frac{1}{2}]) \cdot \mathrm{E}[v_2] = \frac{1}{2} \cdot 34 + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{8}$. The firm's maximum profits from setting $p = V$ would then be $\Pi = V + D_{12} \cdot \phi = \frac{5}{8} + 0 \cdot \phi = \frac{5}{8}$ since $D_{12} = 0$ given $\phi > 0$

from part (ii) of Proposition 14. Since $\phi = 0$ and $p = V = 1$ (noting $V = \mathrm{E}[v_1] + \mathrm{E}[v_2] = \frac{1}{2} + \frac{1}{2} = 1$ given $\phi = 0$) imply $\Pi = 1 > \frac{5}{8}$, $\phi^* = 0$ and $p^* = 1$ must be optimal with account freezing. ■

### C.23 Proof of Proposition 16

*Part (i).* With account freezing, $\Pi = 1$ must always hold from Proposition 15. Without account freezing, $\Pi = 1$ is always attainable by setting $\phi = 0$ and $p = 1$; therefore, $\Pi \geq 1$ without account freezing. The result then follows by comparing $\Pi$ with and without account freezing.

*Part (ii).* With account freezing, $CS = 0$ must always hold from Proposition 15. Letting $\widetilde{CS}$ denote a consumer's subjective expectation of $CS$ (based on $\widetilde{\alpha}$ instead of $\alpha$), $\widetilde{CS} = CS = 0$ must hold — i.e. consumers (correctly) expect $CS = 0$ — since consumers have accurate initial perceptions of their future consumption and tracking behavior (i.e. $\widetilde{D_1} = D_1 = \widetilde{D}_2 = D_2 = 1$ and $\widetilde{D}_\tau = D_\tau = 0$) in this (zero-penalty) equilibrium with account freezing. Without account freezing, consumers would never expect $\widetilde{CS} < 0$ because $\widetilde{CS} = CS = 0$ can always be realized by simply choosing not to subscribe. Thus, $\widetilde{CS} \geq 0$ without account freezing. The result then follows by comparing $\widetilde{CS}$ with and without account freezing.

*Part (iii).* With account freezing, expected total welfare is given by $\Pi + CS = 1 + 0 = 1$, as implied by Proposition 15. Noting that payments of $p$ and $\phi$ only amount to transfers that do not affect total welfare, without account freezing total welfare can be expressed as $\Pi + CS = D_1 \cdot \mathrm{E}[v_1|d_1 = 1] + D_2 \cdot \mathrm{E}[v_2|d_2 = 1] - k \cdot D_\tau \leq D_1 \cdot \mathrm{E}[v_1|d_1 = 1] + D_2 \cdot \mathrm{E}[v_2|d_2 = 1] \leq \mathrm{E}[v_1|d_1 = 1] + \mathrm{E}[v_2|d_2 = 1] = \frac{1}{2} + \frac{1}{2} = 1$. Thus, total welfare must satisfy $\Pi + CS \leq 1$ without account freezing. The result then follows by comparing $\Pi + CS$ with and without account freezing. ■

## D Derivations in Numerical Examples

### D.1 Derivations for Example 1

Given $\alpha = \frac{9}{10}$, $\widetilde{\alpha} = \frac{25}{28}$, $k = \frac{13}{100}$, and $\lambda = 1$, we can use (3) and (4) to compute $\phi_\tau(\alpha, k) = 2$ and $\phi_\tau(\widetilde{\alpha}, k) = \frac{2(70-\sqrt{3101})}{15} \approx 1.908$. Letting $\phi_0 \equiv \min\{\phi : \mathrm{E}[d_2|d_1 = 1, d_\tau = 0, \widetilde{\alpha}]\}$ denote the minimum penalty fee at which a consumer would initially expect, based on her perceived forgetfulness through $\widetilde{\alpha}$, to never consume at $t = 2$ given that she consumes at $t = 1$ and does not use consumption tracking, we can then compute $\phi_0 = \left\{\phi : \frac{(1-\phi)^2 - \widetilde{\alpha}^2\phi^2}{2}\right\} = \frac{28(28-\sqrt{466})}{159} \approx 1.1293$.

With these calculations, we can see that there are four distinct regions in which the firm's profits will depend on $\phi$ in a distinct way.

*Region 1:* $0 \le \phi \le \phi_0$. In this region, $D_1(\phi|\widetilde{\alpha}) = \frac{1-\widetilde{\alpha}^2\phi^2}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 1-(1-\alpha)\phi$. Noting $p(\phi|\widetilde{\alpha}) = \frac{1+D_1^2(\phi|\widetilde{\alpha})}{2}$, we can then calculate $\Pi(\phi|0 \le \phi \le \phi_0) = p(\phi|\widetilde{\alpha}) + D_1(\phi|\widetilde{\alpha}) \cdot D_2(\phi|\alpha, d_1 = 1) \cdot \phi = 1 - \phi^2 \cdot \frac{12{,}258{,}624 - 2{,}458{,}624\phi + 122{,}907\phi^2}{24{,}586{,}240}$, which is decreasing for all $\phi \in [0, \phi_0]$ and is therefore maximized within region 1 at $\phi = 0$, in which case $\Pi = 1$.

*Region 2:* $\phi_0 < \phi \le \phi_\tau(\widetilde{\alpha}, k)$. In this region, $D_1 = 0$, $D_2 = 1$, $p = \frac{1+D_1^2}{2} = \frac{1}{2}$, and $\Pi = p = \frac{1}{2}$.

*Region 3:* $\phi_\tau(\widetilde{\alpha}, k) < \phi \le \phi_\tau(\alpha, k)$. In this region, $D_1(\phi|k) = \frac{1-2k}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 1-(1-\alpha)\phi$. Noting $p(\phi|k) = \frac{1+D_1^2(\phi|k)}{2}$, we can then calculate $\Pi(\phi|\phi_\tau(\widetilde{\alpha}, k) < \phi \le \phi_\tau(\alpha, k)) = p(\phi|k) + D_1(\phi|k) \cdot D_2(\phi|\alpha, d_1 = 1) \cdot \phi = \frac{11{,}369 + 7{,}400\phi - 740\phi^2}{20{,}000}$, which is increasing for all $\phi \in (\phi_\tau(\widetilde{\alpha}, k), \phi_\tau(\alpha, k)]$ and is therefore maximized within region 3 at $\phi = \phi_\tau(\alpha, k) = 2$, in which case $\Pi = \frac{23{,}209}{20{,}000}$ and $p = \frac{11{,}369}{20{,}000}$.

*Region 4:* $\phi > \phi_\tau(\alpha, k)$. In this region, $D_1(\phi|k) = \frac{1-2k}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 0$. Noting $p(\phi|k) = \frac{1+D_1^2(\phi|k)}{2}$, we can then calculate $\Pi(\phi|\phi > \phi_\tau(\alpha, k)) = p(\phi|k) = \frac{11{,}369}{20{,}000}$.

By comparing the maximum profit expressions within each region, we can see that the maximum profits are attained in region 3, implying $\phi^* = 2$, $p^* = \frac{11{,}369}{20{,}000}$, and $\Pi = \frac{23{,}209}{20{,}000}$. In turn, we can also verify that, in this equilibrium, $CS = D_1 \cdot \mathrm{E}[v_1|d_1 = 1] + D_2 \cdot \mathrm{E}[v_2|d_2 = 1] - \Pi - D_\tau \cdot k = \left(\frac{1-2k}{2}\right)\left(\frac{3+2k}{4}\right) + \left[\left(\frac{1-2k}{2}\right)(1-(1-\alpha)\phi^*)\left(\frac{1+(1-\alpha)\phi^*}{2}\right) + \left(\frac{1+2k}{2}\right) \cdot \frac{1}{2}\right] - \Pi - \frac{1-2k}{2} \cdot k = -\frac{3{,}663}{10{,}000}$.

Lastly, from Proposition 2, we can confirm $\alpha = \frac{9}{10} < \frac{8+\max\{3\widetilde{\alpha}-1, 0\}^2}{12} = \frac{2{,}827}{3{,}136}$, implying $\phi^*_{NT} = 0$, $p^*_{NT} = 1$, $\Pi_{NT} = 1$, and $CS_{NT} = 0$.

### D.2 Derivations for Example 2

Given $\alpha = \frac{9}{10}$, $\widetilde{\alpha} = \frac{8}{9}$, $k = \frac{13}{100}$, and $\lambda = 1$, we can use (3) and (4) to compute $\phi_\tau(\alpha, k) = 2$ and $\phi_\tau(\widetilde{\alpha}, k) = \frac{3(30-\sqrt{566})}{10} \approx 1.863$. We can also compute $\phi_0 = \left\{\phi : \frac{(1-\phi)^2 - \widetilde{\alpha}^2\phi^2}{2}\right\} = \frac{9(9-\sqrt{47})}{17} \approx 1.13524$.

With these calculations, we can see that there are four distinct regions in which the firm's profits will depend on $\phi$ in a distinct way.

*Region 1:* $0 \leq \phi \leq \phi_0$. In this region, $D_1(\phi|\widetilde{\alpha}) = \frac{1-\widetilde{\alpha}^2\phi^2}{2}$. Noting $p(\phi|\widetilde{\alpha}) = \frac{1+D_1^2(\phi|\widetilde{\alpha})}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 1-(1-\alpha)\phi$, we can then calculate $\Pi(\phi|0 \leq \phi \leq \phi_0) = p(\phi|\widetilde{\alpha}) + D_1(\phi|\widetilde{\alpha}) \cdot D_2(\phi|\alpha, d_1 = 1) \cdot \phi = 1 - \phi^2 \cdot \frac{129{,}924-26{,}244\phi+1{,}309\phi^2}{262{,}440}$, which is decreasing for all $\phi \in [0, \phi_0]$ and is therefore maximized within region 1 at $\phi = 0$, in which case $\Pi = 1$.

*Region 2:* $\phi_0 < \phi \leq \phi_\tau(\widetilde{\alpha}, k)$. In this region, $D_1 = 0$, $D_2 = 1$, $p = \frac{1+D_1^2}{2} = \frac{1}{2}$, and $\Pi = p = \frac{1}{2}$.

*Region 3:* $\phi_\tau(\widetilde{\alpha}, k) < \phi \leq \phi_\tau(\alpha, k)$. In this region, $D_1(\phi|k) = \frac{1-2k}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 1-(1-\alpha)\phi$. Noting $p(\phi|k) = \frac{1+D_1^2(\phi|k)}{2}$, we can then calculate $\Pi(\phi|\phi_\tau(\widetilde{\alpha}, k) < \phi \leq \phi_\tau(\alpha, k)) = p(\phi|k) + D_1(\phi|k) \cdot D_2(\phi|\alpha, d_1 = 1) \cdot \phi = \frac{11{,}369+7{,}400\phi-740\phi^2}{20{,}000}$, which is increasing for all $\phi \in (\phi_\tau(\widetilde{\alpha}, k), \phi_\tau(\alpha, k)]$ and is therefore maximized within region 3 at $\phi = \phi_\tau(\alpha, k) = 2$, in which case $\Pi = \frac{23{,}209}{20{,}000}$ and $p = \frac{11{,}369}{20{,}000}$.

*Region 4:* $\phi > \phi_\tau(\alpha, k)$. In this region, $D_1(\phi|k) = \frac{1-2k}{2}$ and $D_2(\phi|\alpha, d_1 = 1) = 0$. Noting $p(\phi|k) = \frac{1+D_1^2(\phi|k)}{2}$, we can then calculate $\Pi(\phi|\phi > \phi_\tau(\alpha, k)) = p(\phi|k) = \frac{11{,}369}{20{,}000}$.

By comparing the maximum profit expressions within each region, we can see that the maximum profits are attained in region 3, implying $\phi^* = 2$, $p^* = \frac{11{,}369}{20{,}000}$, and $\Pi = \frac{23{,}209}{20{,}000}$. In turn, we can also verify that, in this equilibrium, $CS = D_1 \cdot \mathrm{E}[v_1|d_1 = 1] + D_2 \cdot \mathrm{E}[v_2|d_2 = 1] - \Pi - D_\tau \cdot k = \left(\frac{1-2k}{2}\right)\left(\frac{3+2k}{4}\right) + \left[\left(\frac{1-2k}{2}\right)(1-(1-\alpha)\phi^*)\left(\frac{1+(1-\alpha)\phi^*}{2}\right) + \left(\frac{1+2k}{2}\right) \cdot \frac{1}{2}\right] - \Pi - \frac{1-2k}{2} \cdot k = -\frac{3{,}663}{10{,}000}$.

Lastly, from Proposition 2, we can confirm $\alpha = \frac{9}{10} > \frac{8+\max\{3\widetilde{\alpha}-1, 0\}^2}{12} = \frac{97}{108}$, implying $\phi_{NT}^* = \frac{1}{1-\widetilde{\alpha}} = 9$, $p_{NT}^* = \frac{5}{8}$, $\Pi_{NT} = \frac{43}{40}$, and $CS_{NT} = -\frac{161}{400}$.

### D.3 Derivations for Example 3

In the limit as $\lambda \to 0$, Example 3 is equivalent to Example 1, where the high tracking cost consumer segment in Example 3 would correspond to "trackers" in Example 1. In light of this, all equilibrium values are readily verified. With sufficiently small $\lambda$, $\phi^*$ and $p^*$ are not affected. Meanwhile, a consumer with $k = k^\ell = 0$ who subscribes to the service and consumes at $t = 1$ will always use consumption tracking and always expect to use consumption tracking since $k^\ell = 0 < k_\tau(\alpha, \phi^*) =$

$\frac{13}{100} < k_\tau(\widetilde{\alpha}, \phi^*) = \frac{27}{196}$ from (3). In turn, a consumer with $k = k^\ell = 0$ will consume at $t = 1$ if and only if $v_1 > \frac{1}{2}$, while consuming at $t = 2$ if and only if $d_1 = 0$. This implies that the perceived and actual valuation of the service for a consumer with $k = k^\ell = 0$ is given by $V = \frac{5}{8}$, and since $V = \frac{5}{8} > p^*$, the consumer will subscribe to the service in equilibrium while expecting and actually choosing to track their consumption with probability $1/2$. This implies $D_\tau = \frac{\lambda}{2}$ and $\widetilde{D}_\tau = \frac{\lambda}{2} + \frac{(1-\lambda)37}{100} = \frac{37+13\lambda}{100}$. Next, the expected surplus to a consumer with $k = k^\ell = 0$ is $V - p^* = \frac{5}{8} - \frac{11{,}369}{20{,}000} = \frac{113}{2{,}000}$, implying total consumer surplus is given by $CS = \lambda \cdot \frac{113}{2{,}000} + (1-\lambda) \cdot \frac{-7{,}326}{20{,}000} = \frac{-7{,}326+8{,}457\lambda}{20{,}000} < 0$ for sufficiently small $\lambda > 0$. Lastly, the expected profit from a consumer with $k = k^\ell = 0$ is $p^* = \frac{11{,}369}{20{,}000}$, implying total expected profit is $\Pi = \lambda \cdot \frac{11{,}369}{20{,}000} + (1-\lambda) \cdot \frac{23{,}209}{20{,}000} = \frac{23{,}209-11{,}840\lambda}{20{,}000} > \Pi_{NT} = 1$ for sufficiently small $\lambda > 0$.